Identification of Induction Motor Parameters from Transient Stator Current Measurements

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Abstract—This paper describes three methods for estimating the lumped model parameters of an induction motor using startup transient data. A three-phase balanced induction motor is assumed. Measurements of the stator currents and voltages are required for the identification procedure, but no measurements from the motor shaft are needed. The first method presented applies simple models with limited temporal domains of validity and obtains parameter estimates by extrapolating the model error bias to zero. This method does not minimize any specific error criterion and is presented as a means of finding a good initial guess for a conventional iterative maximum-likelihood or least-squares estimator. The second method presented minimizes equation errors in the induction motor model in the least-square sense using a Levenburg–Marquardt iteration. The third identification method is a continuation of the Levenburg–Marquardt method, motivated by observed properties of some pathological loss functions. The third method minimizes errors in the observations in the least-squared sense and is, therefore, a maximum-likelihood estimator under appropriate conditions of normality. The performance of the identification schemes is demonstrated with both simulated and measured data, and parameters obtained using the methods are compared with parameters obtained from standard tests.

Index Terms—Parameter estimation, induction motors.

I. INTRODUCTION

THIS PAPER presents techniques for estimating the electrical and mechanical model parameters of an induction motor. The methods described here are applicable to machine characterization, diagnostic, and control applications. The tools in this paper are unusual because, used together, they can determine motor model parameters without an initial guess and with limited observation of only the stator terminal waveforms. The prior work reviewed in the following paragraphs covers a broad range of induction motor state and parameter estimation problems, presented from various perspectives. We anticipate that the methods given in this paper could be applied in many of these problems.

The problems of estimating the parameters and states of a motor have been attacked from a variety of different perspec-

tives; see [1]–[3] for interesting summaries. A variety of online and off-line methods have been proposed for determining the speed of an induction motor rotor. In [4], an extended Kalman filter is used to estimate speed. The authors of [5] and [6] estimate speed as a parameter of the induction machine model, given *a priori* knowledge of the machine electrical parameters. In [7]–[10], the authors identify steady-state slip using a variety of estimators. Dynamic slip estimators are presented in [11] and [12]. A position estimator is described in [13]. All of these methods assume knowledge of most or all of the electrical parameters. In [14]–[16], off-line techniques for determining motor model parameters are presented. All of these methods require data in addition to knowledge of the stator electrical excitation and may even require disassembly of the machine. On-line parameter estimation techniques such as those presented in [13] and [17]–[20] generally estimate a limited subset of the model parameters given knowledge of the remaining parameters. Many of these methods are focused on tracking the rotor resistance or time constant for field-oriented control applications. In [2] and [21]–[24], recursive estimation of both the model parameters and the mechanical state (rotor speed) using transient measurements is considered. These authors fit the induction motor problem to the context of recursive estimation by assuming a separation of electrical and mechanical time constants and treating the mechanical slip $s(t)$ as a parameter. Although the recursive estimation paradigm seems promising because of difficulties like the sensitivity of the rotor resistance to temperature, these techniques require good *a priori* knowledge (better than 10% accuracy) of most of the parameters for successful performance.

A common thread among these induction machine parameter and state estimation problems is the need for either a good initial guess or accurate knowledge of machine parameters. The methods presented in this paper, which are formally off-line, could supplement many of the off- and on-line schemes above in practical engineering applications. For example, a controller designed around one of the on-line state estimators described above could, on installation with an unknown motor, apply a "parameter acquisition" test using our methods to determine parameters or initial guesses required for the state estimator.

Another exciting application area for the methods presented in this paper is machine diagnostics. In [25]–[27], the authors discuss the feasibility of using electrical stator measurements to detect broken rotor bars. Measurements of both electrical and mechanical variables taken from several

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steady-state operating points are used to determine the model parameters. Potentially, our methods could perform a similar function using only stator electrical measurements. In [28] and [29], a transient event detector for nonintrusive load monitoring was introduced that can determine in real time the operating schedule of the individual loads at a target site. This determination is made strictly from observations of transients at the electric utility service entry. In [30], a scheme was presented for exciting and identifying the local impedances in a power delivery network. By combining these tools with a load model parameter estimator, where machine parameters are extracted from observed transients, it might be possible to create a device that could, for example, nonintrusively monitor load health and performance. This paper focuses on induction machines because of their prevalence and industrial importance. However, the techniques presented here could, in principle, be extended to diagnostic applications for other load classes.

II. INDUCTION MOTOR MODEL

In this paper, we consider a three-phase, balanced, singly excited induction motor described by the model presented in [31]. The model expressed in dq currents and voltages is

$$A\begin{pmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{pmatrix} = \begin{pmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{pmatrix} \quad (1)$$

where

$$A = \begin{pmatrix} r_s + X_{ss}\frac{p}{\omega} & X_{ss} & X_m\frac{p}{\omega} & X_m \\ -X_{ss} & r_s + X_{ss}\frac{p}{\omega} & -X_m & X_m\frac{p}{\omega} \\ X_m\frac{p}{\omega} & sX_m & r_r + X_{ss}\frac{p}{\omega} & sX_{ss} \\ -sX_m & X_m\frac{p}{\omega} & -sX_{ss} & r_r + X_{ss}\frac{p}{\omega} \end{pmatrix}. \quad (2)$$

In (2), the inductances appear as impedances at the base electrical frequency, e.g., 60 Hz (377 rad/s). For example, the magnetizing impedance X_m at 60 Hz is

$$X_m = \omega M = 120\pi \cdot M \quad (3)$$

where M is the magnetizing inductance. Also, in (2), $X_{ss} = X_m + X_l$, where X_l is a leakage impedance. All rotor quantities are as referred to the stator. The currents and voltages in (1) are expressed in the synchronously rotating dq frame. Details on transformation of quantities in the laboratory frame to and from the rotating frame can be found in [31]. The assumption of single excitation implies that $v_{dr} = v_{qr} = 0$. That is, the rotor is excited only by the stator. Table I lists units and physical interpretations for some of the variables appearing in (2) and throughout. Dimensions for parameters and quantities not listed in Table I are readily inferred from their context.

Equation (1) can be expressed in complex notation using the definitions

$$\begin{aligned} i_s &= i_{qs} + ji_{ds} \\ i_r &= i_{qr} + ji_{dr} \\ v_s &= v_{qs} + jv_{ds} \\ v_r &= v_{qr} + jv_{dr} \end{aligned} \quad (4)$$

TABLE I
SYMBOLS AND UNITS OF BASIC VARIABLES AND PARAMETERS

Symbol	Variable or Parameter	Units
X_m	magnetizing impedance	Ω at ω rad/s
X_l	leakage impedance	Ω at ω rad/s
r_r	rotor resistance	Ω
r_s	stator resistance	Ω
ω	base electrical frequency	rad/s
ω_s	synchronous speed	rad/s
ω_r	rotor speed	rad/s
p	time derivative, $\frac{d}{dt}$	1/s
s	slip	–

where $j = \sqrt{-1}$. In complex variables, (1) is

$$\begin{pmatrix} v_s \\ v_r \end{pmatrix} = \begin{pmatrix} r_s + X_{ss}\left(\frac{p}{\omega} - j\right) & X_m\left(\frac{p}{\omega} - j\right) \\ X_m\left(\frac{p}{\omega} - sj\right) & r_r + X_{ss}\left(\frac{p}{\omega} - sj\right) \end{pmatrix} \begin{pmatrix} i_s \\ i_r \end{pmatrix}. \quad (5)$$

The electromechanical interaction enters (1) and (5) through the slip s,

$$s = \frac{\omega_s - \omega_r}{\omega_s} \quad (6)$$

and the torque

$$T = \frac{3}{2}\frac{P}{2}M(i_{qs}i_{dr} - i_{ds}i_{qr}) \quad (7)$$

where P is the number of poles. The mechanical system attached to the motor determines the interaction between w_r and T and is modeled using ordinary mechanics. For the purposes of this paper, the mechanical load is modeled by an inertia J and linear coefficient of friction B as

$$T = Jp\omega_r + B\omega_r. \quad (8)$$

This model serves as an example, but methods described in this paper are, in principle, not limited to this selection. Alternate parameterizations and choices of state variables are sometimes useful in identifying the induction motor parameters. A reparameterization of the mechanical system combining (6) and (8) is

$$ps = \beta(1 - s) - \frac{T}{J\omega_s}. \quad (9)$$

The new parameter β equals B/J. An alternate set of complex state variables is flux linkages per second,

$$\begin{aligned} \Psi_s &= X_{ss}i_s + X_m i_r \\ \Psi_r &= X_{ss}i_r + X_m i_s. \end{aligned} \quad (10)$$

Similarly, it is sometimes useful to eliminate the impedances X_m and X_{ss} in favor of

$$\begin{aligned} Y_m &= \frac{X_m}{X_{ss}^2 - X_m^2} \\ Y_{ss} &= \frac{X_{ss}}{X_{ss}^2 - X_m^2} \end{aligned} \quad (11)$$

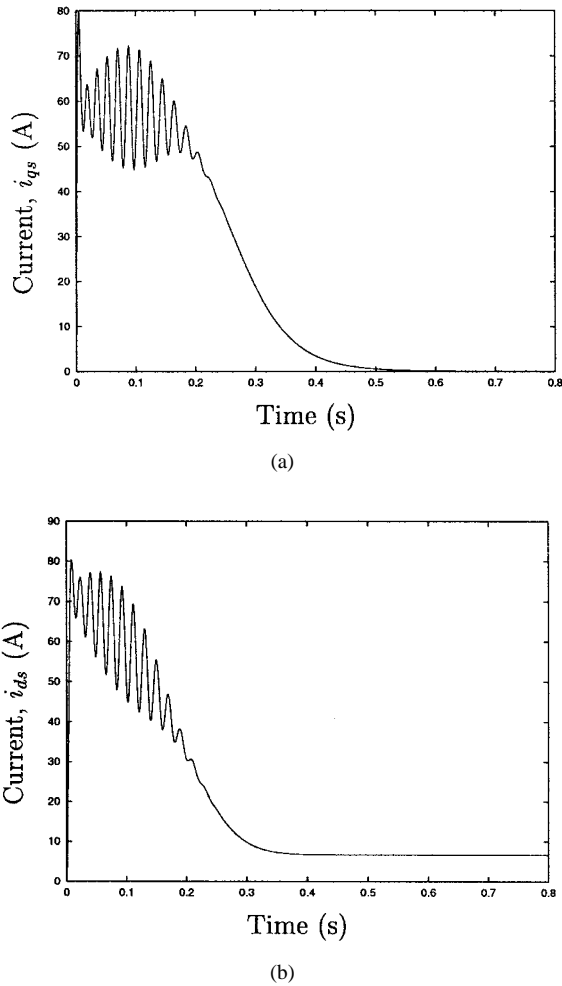which have the dimensions of admittance.

Fig. 1. Motor starting transient; i_{qs} and i_{ds}.

A typical simulation of the free acceleration of an induction motor follows directly from the model. For example, using the parameters

$$X_m = 26.13$$
$$X_l = 0.754$$
$$r_r = 0.816$$
$$r_s = 0.435$$
$$J = 0.089$$
$$B = 0 \qquad (12)$$

with an excitation of 220 V line to line, the results in Fig. 1 are obtained by simulating (1) and (8). These parameters describe a 3-hp rated motor and can be found in [31].

III. PARAMETER ESTIMATION

If the slip s and the complex state i_r were available directly for measurement, the induction motor parameter estimation problem given (1) would follow directly from standard techniques. Since (1) is a differential equation, one possible solution technique would be to eliminate the time derivative p with an operator substitution and solve the resulting linear least-squares problem [32].

Measurements of i_{qs} and i_{ds} or the laboratory frame currents i_{as}, i_{bs}, and i_{cs} are the only data available in the nonintrusive load monitoring scenario [28] and are likely the most convenient set of measurements to make in general. The following parameter estimation schemes use only measurements of the stator currents i_{qs} and i_{ds} and the stator voltages v_{qs} and v_{ds}. Measurements of the source voltages v_{qs} and v_{ds} may not be necessary if the electrical supply has been characterized as in [30].

A. Extrapolative Method

The extrapolative method developed here is motivated by the idea of quickly "eyeballing" data to obtain reasonably accurate parameter estimates. These parameter estimates are not likely to be least-squares solutions, but might be sufficiently accurate for some applications. In situations where a specific error criteria such as least squares must be minimized by an iterative routine, the method presented here could dramatically increase computational efficiency by supplying a good initial guess.

The philosophy of the method is to decompose a transient described by a complicated model into smaller domains described by simple, easy to identify models. For example, from circuit intuition, it is clear that under the condition $s = 1$, which exists briefly at $t = 0$ due to inertial confinement of the rotor,

$$i_r = -i_s. \qquad (13)$$

The induction motor may be thought of as a transformer with a shorted secondary in this state. Substituting (13) into the complex induction motor model (5) yields

$$v_s = (r_s + r_r)i_s + 2X_l\left(\frac{p}{\omega} - j\right)i_s. \qquad (14)$$

Equation (15) is a relatively simple, "high-slip" model with the potential to remove two degrees of freedom from the parameter estimation problem using only i_s and v_s.

Similarly, under conditions of zero slip, $i_r = 0$. In this state, the induction motor is very much like a transformer with an open secondary. Substituting into (5),

$$v_s = r_s i_s + (X_l + X_m)\left(\frac{p}{\omega} - j\right)i_s. \qquad (15)$$

Equation (15) is a "low-slip" model, which might be valid during the steady-state operation of an induction machine loaded with an inertia only.

The operating conditions required to make models (14) and (15) valid represent a small fraction of the data recorded during a typical transient, as shown in Fig. 1. If these models are applied to nearby data at times γ for which the model is *not* valid, the parameter vector estimate $\hat{\theta}$ will exhibit a bias, i.e.,

$$\hat{\theta}(\gamma) = \theta(0) + \beta(\gamma). \qquad (16)$$

If the model is valid at time $\gamma = 0$, then the model error bias $\beta(\gamma) \to 0$ as $\gamma \to 0$. Any bias which might be due to noise is neglected. Assuming that the model error $\beta(\gamma)$ is a well-behaved function, the estimates $\hat{\theta}(\gamma)$ can be modeled with an appropriate interpolating function and

the unbiased estimate $\hat{\theta}(0)$ determined by interpolation or extrapolation as the circumstances dictate. The extrapolation process is essentially Richardson's deferred approach to the limit, described in [33].

For this paper, the extrapolative technique was applied to models (14) and (15) using rational functions for extrapolation [33]. Note that no particular loss function is minimized by this estimation technique. It is quite likely that the estimates are not least-square estimates. On the other hand, no initial guess is required and there is no iteration. This method may significantly reduce the effort and complexity of an iterative estimation method by providing good initial guesses for some subset of the parameters. The method is described in more detail in [34].

B. Equation Error Method

Although some of the states in the induction motor model are not measured in the assumed scenario, the parameters can be found by iterative minimization of a cleverly designed loss function. Since the excitation v_s and the stator currents i_s are known or measured, an estimate of the parameter vector θ is given by a minimization of the form

$$\hat{\theta} = \arg\min_{\theta} V(\theta, i_s, v_s). \tag{17}$$

One approach to formulating a loss function V based on the motor model is to algebraically eliminate the unmeasured quantities. A loss function based on the transformed model, expressed only in terms of measured or known quantities, could then be used to find the desired parameters. For example, in a model given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \tag{18}$$

where x_2 is not measured, if b (in general, an operator) is invertible, then

$$cx_1 + db^{-1}(r_1 - ax_1) = r_2. \tag{19}$$

Equation (19) has the desired property of containing only x_1, and standard techniques [32] could be applied to finding the parameters in the operators a–d. If b is not convenient to invert, it might still be possible to eliminate x_2. Applying b and d to (18) yields

$$\begin{aligned} dax_1 + dbx_2 &= dr_1 \\ bcx_1 + bdx_2 &= br_2. \end{aligned} \tag{20}$$

If the commutator (Lie bracket) $[b, d] = bd - db$ is equal to zero, then the term bdx_2 can be isolated in one equation and substituted in the other, eliminating x_2. If $[b, d] = 0$, then

$$dax_1 + br_1 - bcx_1 = dr_2. \tag{21}$$

The complex induction motor model (5) has the same form as (18). Eliminating i_r by inverting one of the operators acting on i_r does not seem promising. Both operators acting on i_r have a zero; their corresponding inverses have a pole which could introduce internal stability issues. The operators acting on i_r also have a nonzero commutator due to the time variation of

s. To eliminate i_r, one must introduce either errors due to the pole in the inverses, or errors due to the nonzero commutator.

For the equation error method, we estimate i_r and attempt to confine the errors due to the resulting pole. The i_r estimator is derived from (10) and the top row of (5), which can be rewritten as

$$\hat{\Psi}_s = \frac{\omega}{p - \omega j}(v_s - r_s i_s). \tag{22}$$

Recall that ω is the base frequency, usually 120π. Using the definition of Ψ_s

$$\Psi_s = X_m(i_r + i_s) + X_l i_s \tag{23}$$

an estimate of i_r is

$$\hat{i}_r = \frac{1}{X_m}(\hat{\Psi}_s - X_m i_s - X_l i_s). \tag{24}$$

To the extent that (22) can be computed accurately, then, $\hat{i}_r$ is expressed in terms of the parameters and the observations i_s and v_s. Having used half the induction motor model (5) to estimate the rotor currents, the second half provides a useful check of the consistency between current estimates and the data. Under the assumed conditions of single excitation, $v_r = 0$. An estimate for the rotor voltage is

$$\hat{v}_r = X_m\left(\frac{p}{\omega} - \hat{s}j\right)i_s + \left(r_r + (X_m + X_l)\left(\frac{p}{\omega} - \hat{s}j\right)\right)\hat{i}_r. \tag{25}$$

If the parameters are incorrect or if $\hat{i}_r$ is not equal to the unmeasurable i_r, there will be some error ϵ

$$\epsilon = \hat{v}_r - v_r. \tag{26}$$

The error ϵ will be a combination of the errors due to the pole at ω in (22) and the error due to the current parameter estimate.[1] However, the errors due to the observer pole will occur locally in the frequency domain around the pole frequency ω. Although the errors due to the observer pole are modulated by the time variation in the slip, the slip is relatively slow (given reasonable mechanical loads) in comparison to the observer pole frequency. Note that it is more convenient to use the top row of (5) for the observer because the pole is located at the fixed frequency ω. The errors due to parameter mismatch will occur at lower frequencies. For example, if r_r is off by δ, the error ϵ will be a "copy" of the mostly low-pass rotor current, i.e., $\epsilon = \delta \hat{i}_r$.

The solution is to minimize $\epsilon^T \epsilon$ in the frequency domain at those frequencies where the artifacts introduced by the observer pole have no effect. The advantage of using the frequency domain is that the minimization can be applied selectively to the errors that are due to parameter mismatch, ignoring the observer pole artifacts. In practice, spectral leakage due to finite data set length [32] causes errors due to the observer pole to influence the errors due to parameter mismatch. Thus, as a practical measure, the residuals ϵ are weighted with a Blackman window [35].

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[1] Note that, according to the definition of the error in (26), the estimated parameters will *not* be "least-square parameters" in terms of the observations i_s.

Note that if the slip is not measured, the estimate $\hat{s}$ required for (25) can be obtained from (24) and the mechanical model. Unfortunately, the slight errors introduced by the observer pole tend to accumulate when integrating the mechanical model to obtain a slip estimate. To work around this, the mechanical model is reparameterized, replacing the inertia and drag parameters with s_0 and t_0, where s_0 is the value of the slip at some time t_0, i.e., $s_0 = s(t_0)$. This is a convenient reparameterization, because the steady-state speed and approximate duration of the startup transient are likely known to fair accuracy *a priori*. Obtaining the desired slip estimate $\hat{s}(t)$ under the new parameterization is then a boundary value problem, which can be solved by a shooting method [33], [34]. This approach would require reformulation for load models other than (8).

Parameter estimates were obtained by minimizing low-frequency (with respect to ω) components of the residual ϵ using a Levenburg–Marquardt iteration derived substantially from [33].

C. General Identification Method

Another possible candidate for the loss function $V(\theta, i_s, v_s)$ can be developed by setting i'_s to the results of a simulation using the parameters θ and the excitation v_s. Then, the loss function is the squared error between the observed and simulated currents, i.e.,

$$V(\theta, i_s, v_s) = (i_s - i'_s(v_s, \theta))^T (i_s - i'_s(v_s, \theta)). \quad (27)$$

Disadvantages to this approach are that, for a Jacobian-based minimization method, the Jacobian must be assembled using finite differences, evaluations of the loss function are likely to be computationally costly, and the unmodified loss function is likely to have many local minima, particularly as the number of unmeasured states increases. However, (27) is extremely attractive because of its simplicity and generality; potentially, one need only specify a model for a system to identify its parameters. By considering the properties of the loss function associated with a simple system, we motivate an algorithm that ameliorates the disadvantages inherent in this general approach.

Consider a possibly nonlinear state-space model with state x, input u, and outputs or measurements y given by

$$px = F(x, u) \quad (28)$$
$$y = G(x, u) \quad (29)$$

where parameters θ are embedded in F and G. Assume that the structures of F and G are known *a priori*, and we would like to find the parameters $\hat{\theta}$ that minimize the least-squared error between the measurements y and the predictions $\hat{y}$, i.e., the loss function is

$$V(\theta) = (y - \hat{y})^T (y - \hat{y}). \quad (30)$$

The induction motor, and many other problems, fit this context. The problem is quite difficult even for the simplest F and G. For example, let

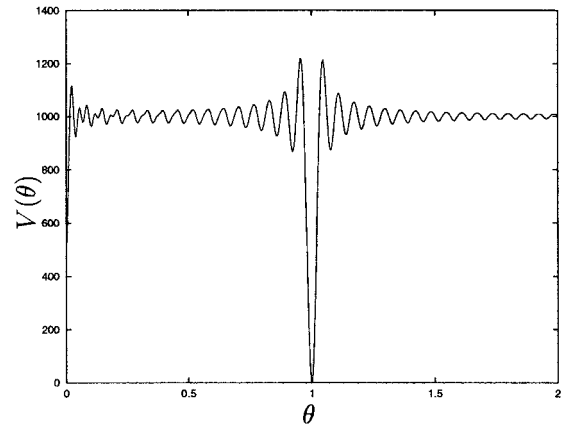$$F(x, u) = \begin{pmatrix} 0 & \beta \\ -\beta & 0 \end{pmatrix} x \quad (31)$$

Fig. 2. Loss function $V(\theta)$ defined in (13).

and

$$G(x, u) = (1 \quad 0)x \quad (32)$$

then find an estimate $\hat{\theta}$ of β given measurements of $\sin(\beta t)$. For N observations sampled with period T, the loss function $V(\theta)$ is

$$V(\theta) = \sum_{i=1}^{N} (\sin(\beta iT) - \sin(\theta iT))^2. \quad (33)$$

In Fig. 2, $V(\theta)$ is plotted for $\theta \in [0, 2]$ with $T = .1$, $N = 1024$ and $\beta = 1$. The numerous local minima in $V(\theta)$ imply that, for conventional methods to converge on the correct estimate $\hat{\theta} = \omega$, the initial guess must be extremely good. Fig. 2 demonstrates that the difficulty of an identification problem is related less to the input–output complexity of the system than to the way the parameters are embedded in the measured signals. Nonlinear systems can result in linear identification problems; (31) is an example of a linear system were the parameters are embedded nonlinearly in the response. Nonlinear functions often exhibit simplified behavior in restricted domains; this is the basis of small-signal techniques common in circuit design and simulation. By analogy, one way to simplify the loss function is to restrict the length N of the interval over which (33) is evaluated. In effect, the algorithm temporarily discards data to obtain a loss surface with simpler topology. The effect is shown graphically in Fig. 3, where the normalized loss function $V(\theta, N)/N$ is plotted over the θ, N plane. Note that, for small N, the valley leading to the global minimum at $\theta = 1$ is wider, implying that convergence will succeed for a wider range of initial guesses θ'. One interpretation is that the interval N must be sufficiently small so that the difference $\sin(\theta t) - \sin(\beta t)$ is functionally related to the error in the parameters $\theta - \beta$. To put this observation to work, coarse estimates should be refined using small N; as the estimate improves, N should be expanded to include the entire data set.

The observation that the loss function topology can be simplified by restricting the domain applies to systems more complicated than (31). Any system where perturbations in the parameters, on the order of the error in the parameters, result in monotonic behavior of the loss function over a
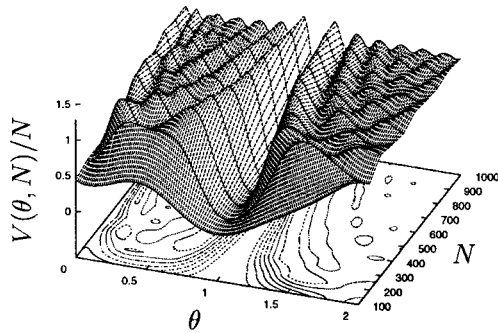
Fig. 3.   Loss function $V(\theta, N)/N$.

restricted number of points N' is amenable to the method. This condition includes many, but not all, nonlinear problems. Note that some responses, such as a step parameterized by a delay, are nonlinear in the parameters and also do not meet the criteria above, but have simple loss functions that do not depend on topological simplification for solution. Taking these two observations into account, we proposed the following algorithm.

1) Choose initial interval N', set $N_{\min} = N'$.
2) Minimize loss function over N'.
3) If minimization is successful,
$$N' = \min(\alpha N', N)$$
$$N_{\min} = N'.$$
4) If minimization is unsuccessful,
$$N' = \max(\beta N', N_{\min}).$$
5) Repeat from 2) until $N' = N$.

For this paper, a growth factor $\alpha = 2$ and a shrinking factor $\beta = .75$ were used. The minimizations displayed little sensitivity to other reasonable choices of these parameters, although the contraints $\alpha > 1$ and $\beta \in (0,1)$ should be observed. Many options exist for determining the success of the individual minimizations [32]. There is no unequivocal solution. One must adopt a criterion that decides, preferably with some statistical insight, whether or not the residual can be further reduced by restricting the domain of the loss function and attempting another minimization. Note, however, that the criterion chosen is critical only when there is high noise or when the response is complicated. For this paper, we found that consideration of the magnitude of the N' normalized residual proved useful. Given some knowledge of the statistical properties of the disturbances, test statistics like the zero-crossing test or the Kolmogorov–Smirnov test might also be used [32], [33].

Despite that each evaluation of the loss function requires a simulation, the method outlined above is more computationally efficient that one might expect. The initial estimate is refined using relatively few points and, hence, a shorter simulation. When the number of points N' is increased, the simulations become more expensive, but the number of iterations decreases because the parameter estimate is hopefully close to optimal.

When applying the method to the induction motor model, the loss function was calculated by comparison of adaptive step-size Runge–Kutta simulations of the model to the measured data, as in (27). The minimizations were performed by

TABLE II
EXTRAPOLATIVE METHOD ESTIMATES, SIMULATED DATA

Motor		True Parameters	Estimated Parameters
3 hp 220 V	X_m	26.13	26.144
	X_l	.754	.739
	r_r	.816	.808
	r_s	.435	.434
50 hp 460 V	X_m	13.08	13.38
	X_l	.302	.2917
	r_r	.228	.2702
	r_s	.087	.0647
500 hp 2300 V	X_m	54.02	54.10
	X_l	1.206	1.205
	r_r	.187	.472
	r_s	.262	.214
2250 hp 2300 V	X_m	13.04	13.03
	X_l	.226	.231
	r_r	.022	.034
	r_s	.029	.029

the routine LMDIF in MINPACK,[2] which is a finite-difference Levenburg–Marquardt code. The reparameterizations of (9) and (11) were used with the flux linkage per second state variable form of (1).

IV. RESULTS

The induction motor identification methods presented here were tested using both simulated and measured data. Tests using simulated data compare the estimated parameters to the parameters used to create the simulated data. To evaluate the performance of the methods on real data, estimated parameters were used to create simulated transients which are compared to the measured data. No noise was added to the simulated data, and no filtering or smoothing was performed on the real data. Note that validation with simulated data tests the identification procedure in isolation, while validation with measured data tests both the motor model and the identification procedure.

A. Simulated Data

Simulated data were obtained by numerical simulation of (1) for a variety of motor parameter sets. The simulated motors include the 3-, 50-, 500-, and 2250-hp motors used as examples in [31]. Note that the implied test procedure, in particular application of full rated voltage and frequency, is probably not realistic for larger machines. We include the higher power examples to test the methods and demonstrate that, in principle, the minimization methods work for large machines. In practice, somewhat different excitation may need to be used. The methods presented in this paper should be applicable, provided that the voltage excitation is sufficiently rich and known or measured.

1) Extrapolative Method: Results for the extrapolative method are shown in Table II. The column labeled "True Parameters" contains the values used by the simulator to generate a transient. The column labeled "Estimated Parameters" contains the parameters output by the extrapolative

TABLE III
EQUATION ERROR METHOD, SIMULATED DATA

Motor		True Parameters	Initial Guess	Estimated Parameters
3 hp 220 V	X_m	26.13	24.0	26.49
	X_l	.754	1.0	.7521
	r_r	.816	1.0	.7705
	r_s	.435	0.3	.4169
50 hp 460 V	X_m	13.08	10.0	12.99
	X_l	.302	0.5	.2984
	r_r	.228	0.5	.1950
	r_s	.087	.1	.0845
500 hp 2300 V	X_m	54.02	50.0	52.22
	X_l	1.206	1.0	1.207
	r_r	.187	.3	.1829
	r_s	.262	.4	.2643

TABLE IV
GENERALIZED IDENTIFICATION METHOD, SIMULATED DATA

Motor		True Parameters	Initial Guess	Estimated parameters
3 hp 220 V	Y_m	.6537	.5	.6537
	Y_{ss}	.6726	.6	.6726
	r_r	.8160	.5	.8160
	r_s	.4350	.5	.4350
	J	.0890	.01	.0890
50 hp 460 V	Y_m	1.637	1.4	1.637
	Y_{ss}	1.675	1.5	1.675
	r_r	.2280	0.5	.2280
	r_s	.0870	.05	.0870
	J	.8300	1.0	.8300
500 hp 2300 V	Y_m	.4100	1.4	.4100
	Y_{ss}	.4192	1.5	.4192
	r_r	.1870	.5	.1870
	r_s	.2620	.05	.2620
	J	22.80	1.0	22.80
2250 hp 2300 V	Y_m	2.193	1.4	2.193
	Y_{ss}	2.231	1.5	2.231
	r_r	.0220	.05	.0220
	r_s	.0290	.05	.0290
	J	63.87	10.0	63.87

TABLE V
BOILERPLATE DATA FROM TEST INDUCTION MOTOR

Leyland-Faraday Electric Company		
Type: AEEA	Model: LFI-3050	Phase: 3
Hp: 5	Volts: 208-230/460	
Rating: Cont	Cycles: 60	
AmbTemp: 40 C	RPM: 3420-3480	
Frame: 184 T	Service Factor: 1.15	
Amps: 12/6	Nema Design: B	

identification program. Recall that no initial guess is required for the extrapolative method, and that the method is not iterative.

2) Equation Error Method: Table III shows results from the equation error method. Although no particular care was taken in picking initial guesses, informal experimentation revealed that the convergence of the method was rather robust. Rigorous analysis of the convergence properties of the method is left to future work. In Table III, "True Parameters" are the parameters used by the simulator to generate test data. The "Initial Guess" column lists the parameter guess passed to the iterative method. Note that the initial guesses in Table III are uniformly worse than than the results from the extrapolative method listed in Table II. The results in Table III were obtained using the shooting method slip estimation scheme discussed in Section III.A.

3) Generalized Identification Method: The results of the generalized identification method on simulated data are shown in Table IV. Note that the generalized method estimates the mechanical parameters directly instead of estimating the slip. In all cases, for inertially loaded, simulated motors from 3 to 2250 hp, the generalized identification method computed answers that were exact to the number of significant digits in the parameters supplied to the simulation.

B. Measured Data

Real data was obtained from transient tests on a typical three-phase industrial induction motor. The test induction motor was connected to a three-phase 208-V line-to-line 30-A rated service using a solid-state three-phase switch with a programmable firing angle [36]. Voltage and current data during the startup transient were collected on a four-channel Tektronix TDS420A digital storage oscilloscope using isolated Tektronix A6909 voltage and A6303 current probes. Since the firing angle was under computer control, the experiment was assumed to be repeatable. Hence, the current and voltage measurements were actually made in two successive tests. For synchronization of the two tests, one channel of voltage information was stored while collecting the three currents. The oscilloscope was set to trigger from this voltage channel for both current and voltage measurements. Data collected by the oscilloscope was stored on disk and translated to files suitable

for input to the identification procedures. No mechanical load, except for the rotor inertia and windage, was attached to the motor.

In addition to applying the three identification methods to the measured data, standard blocked rotor, dc, and no-load measurement techniques were performed on the test motor. The boilerplate data from the test induction motor is reproduced in Table V.

1) Extrapolative Method: Results for the extrapolative method applied to the measured data set appear in Table VI(a). The estimates are reasonable except for the value of r_s. The estimate of r_s depends on the identification of the "low-slip" model. Since the low-slip model assumes only an inertia as a mechanical load, the failure is not too surprising; the real motor has unmodeled load torque from its internal cooling fan and and bearings. Again, the extrapolative method is a direct method, with no initial guess.

2) Equation Error Method: The measured induction motor data were also analyzed with the equation error method. Parameter estimates are given in Table VI(b). The slip estimate is shown in Fig. 4. Note that the mechanical parameters, although not shown in the table, could be obtained given the estimated slip curve and electrical parameters. The equation error method converged in six iterations. To validate the parameter and slip estimates in Table VI(b), the measured voltage waveforms, parameter, and slip estimates were input to

TABLE VI
ESTIMATED INDUCTION MOTOR PARAMETERS, MEASURED
DATA (a) EXTRAPOLATIVE METHOD (b) EQUATION ERROR
METHOD (c) GENERAL METHOD (d) GENERAL METHOD

Parameter	Estimate
X_l	5.47e-01
X_{ss}	3.95e+01
$r_r + r_s$	7.10e-01
r_s	7.60e+00

(a)

Parameter	Initial Guess	Estimate
X_l	1.00e+00	6.05e-01
X_m	4.50e+01	3.85e+01
r_r	5.00e-01	3.36e-01
r_s	5.00e-01	4.76e-01

(b)

Parameter	Initial Guess	Estimate, Model 1
Y_m	5.00e-01	8.28e-01
Y_{ss}	6.00e-01	8.47e-01
r_r	5.00e-01	3.59e-01
r_s	5.00e-01	4.58e-01
J	1.00e-02	4.12e-02

(c)

Parameter	Initial Guess	Estimate, Model 2
Y_m	9.10e-01	8.27e-01
Y_{ss}	9.20e-01	8.44e-01
r_r	3.55e-01	3.56e-01
r_s	3.55e-01	4.57e-01
J	2.00e-02	4.02e-02
β	0.00e00	2.64e-01

(d)

Fig. 5. Comparison of measured (thin line) and simulated (thick line) dq currents, equation error method.
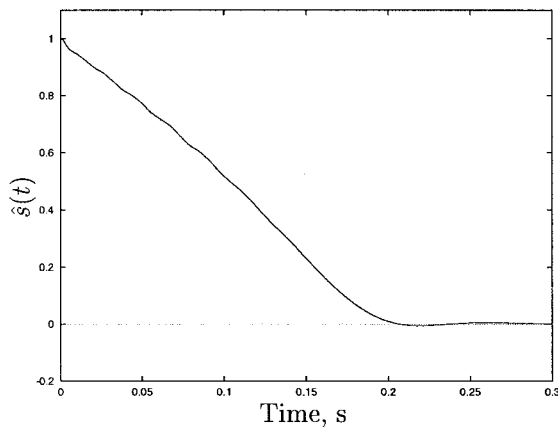

Fig. 4. Estimated slip, equation error method.

a simulator in an attempt to reproduce the measured currents. A comparison of the measured current and the current produced by simulation with the estimated parameters is shown in Fig. 5. This test not only verifies the accuracy of the parameter estimates, it also validates the applicability of the assumed model. It should be noted th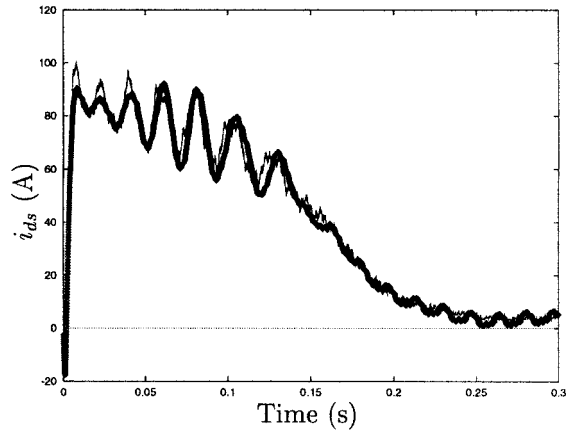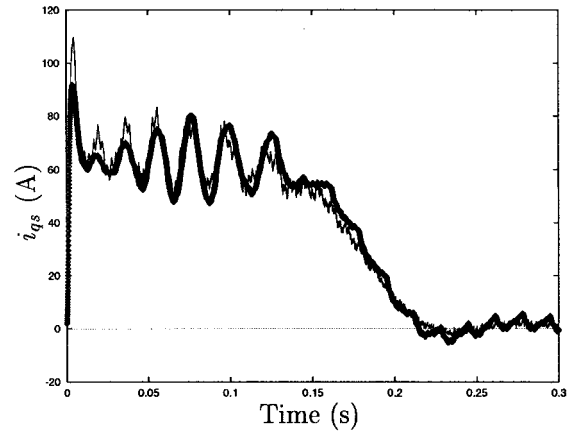at, while a balanced three-phase service was assumed for both identification and validation, the measured voltage distortion was imbalanced. This imbalance may be responsible for some of the "ripple" seen in Fig. 5. The mismatch between measurement and prediction on the initial spike in Figs. 5–7 is likely due to unmodeled magnetic saturation.

3) Generalized Identification Method: The generalized identification method was also applied to the measured data. The data were fit with two different mechanical load models. Model 1 was an inertia only, i.e., β in (9) set to zero. Model 2 included an inertia and linear damping term, as in (9). The flexibility of the general identification method made changing models quite trivial. However, some difficulties were encountered when trying to identify all six parameters of Model 2. In particular, when the generalized method was working on early portions of the transient, the unmodeled magnetic saturation in the initial spike of the transient tended to cause the parameters of the mechanical subsystem to go to unreasonable values. The general method would "stall" on the portion of the transient where the magnetic saturation causes a relatively high error, attempting to adjust the mechanical subsystem parameters to account for the error in the model. This was not a problem with Model 1, presumably because the effects of magnetic saturation could not be accomodated by adjusting the inertia only. To obtain the Model 2 results in Table VI(d), we took a practical approach; instead of

TABLE VII
COMPARISON OF ELECTRICAL PARAMETERS FOR TRANSIENT IDENTIFICATION
METHODS AND STANDARD TEST PROCEDURE

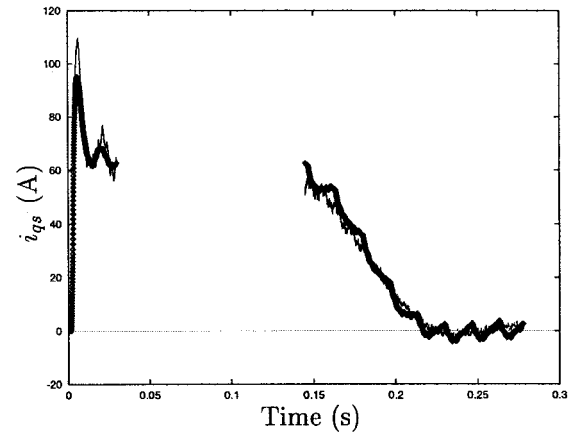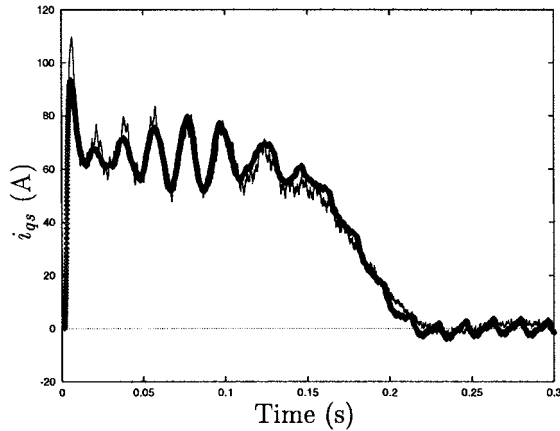| Parameter	Extrapolative Method	Equation Error Method	General Method Model 1	General Method Model 2	Standard Test Method
X_m	3.89e+01	3.85e+01	2.59e+01	2.71e+01	3.5e+01
X_l	5.47e-01	6.05e-01	5.97e-01	5.98e-01	6.8e-01
$r_r + r_s$	7.1e-01	8.12e-01	8.17e-01	8.13e-01	7.9e-01
r_s	7.6e+00	4.76e-01	4.58e-01	4.57e-01	4.7e-01
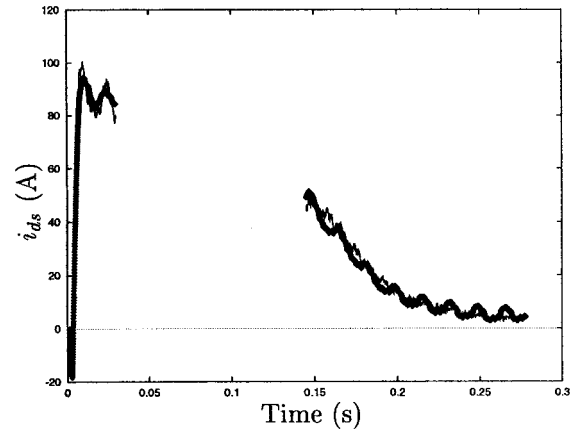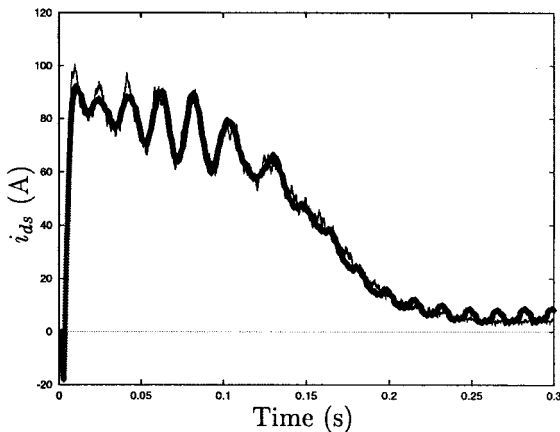
Fig. 6. Comparison of measured (thin line) and simulated (thick line) dq currents, general method.

Fig. 7. Comparison of measured (thin line) and simulated (thick line) dq currents for an incomplete data set, general method.

the aggressive initial guess used for Model 1, we used an initial guess based on the results of the extrapolative method, Table VI(a). The initial guesses for Y_m and Y_{ss} follow directly from the extrapolative method results for X_m and X_{ss} and (11). The initial guess for r_r and r_s was one-half of the extrapolative method estimate for $r_r + r_s$, and the initial estimates for the remaining parameters were made arbitrarily. In addition, for identification of Model 2, the general method was started with a large initial interval N' to lessen the relative contribution of the model error to the residual. Parameters for Models 1 and 2 appear in Tables VI(c) and (d). Measured and simulated currents, using Model 1, are compared in Fig. 6. In Fig. 7, we demonstrate that the general method is applicable to analysis of partial data sets. In Fig. 7, the middle portion

of the measured transient has been eliminated, but the method still converges, without any code changes.

4) Standard Test Method: Standard blocked rotor, no-load, and dc measurements were made on the test machine, as in [14]. The results appear in Table VII, where they are compared to equivalent parameters determined by the other methods. In Table VII, recall that the different methods minimize different error criteria. The choice of error criterion is reflected in the parameter estimates.

V. CONCLUSION

In this paper, we have presented three techniques for finding the parameters of an induction motor given transient stator-

current waveforms. These algorithms were demonstrated on both simulated and measured data.

The extrapolative method performed extremely well in its intended role, to provide an initial guess. On simulated data, the results are good. However, the utility of the method is especially clear in Table VII. With the exception of r_s, the estimates from the extrapolative method are reasonably close to the parameters found by the other methods. Although the extrapolative method might be regarded as *ad hoc*, there is little reason to prefer the potentially highly inefficient minimization of a formal loss function when parameters are far from the minimum. An initial guess provided by the extrapolative method was used successfully to help the generalized method overcome the effects of unmodeled magnetic saturation when fitting Model 2 to the data.

The equation error method performed well on both simulated and measured data. Unfortunately, considerable effort was required to formulate estimators and error criterion that would have low sensitivity to artifacts such as the observer pole. Furthermore, the error criterion does not allow a convenient statistical interpretation of the final estimate. The method could be applied to other models, but only with considerable effort.

The generalized method proved to be outstanding. The loss function is a natural one, the model is easily modified, and performance is comparable to the other methods. We have applied the generalized method to other problems and models and found that its performance is uniformly good. The ability of the general method to converge on incomplete data sets is especially promising, since it implies that a particular load could be identified from features extracted from a mosaic of transients from other unrelated loads. Although there were some difficulties identifying Model 2 with the generalized method, these difficulties were the result of shortcomings in the motor model and were corrected by using a better initial guess and a longer initial interval N'. Overall, the general method seems to minimize operator effort in identification problems.

It seems clear that these methods are directly applicable to a variety of induction motor parameter estimation problems, ranging from characterization for controller design to noninvasive diagnostics like broken rotor bar detection. We suspect that the generalized method will find further applications.

REFERENCES

[1] J. Lang, G. Verghese, and M. Ilic-Spong, "Opportunities in estimation and control of electrical machines," in *Proc. 1986 IEEE Conf. Decision and Control*, pp. 379–387.

[2] M. Vélez-Reyes, "Speed and parameter estimation for induction machines," M.S. thesis, Dep. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, May 1988.

[3] J. Lang and G. Verghese, "Adaptive estimation of state and parameters in electrical machines," in *Proc. Int. Workshop Microcomputer Control of Electric Drives*, Trieste, Italy, July 3–4, 1989, pp. C1.1–C1.7.

[4] F. Hillenbrand, "A method for determining the speed and rotor flux of the asynchronous machine by measuring the terminal quantities only," in *Proc. 1984 IFAC Symp. Control in Power Electronics and Electrical Drives*, pp. 55–62.

[5] B. de Fornel, J. Farines, and J. Hapiot, "Numerical estimation of the speed of an asynchronous machine supplied by a static converter," in *Conf. Rec. 1979 IEEE-IAS Annu. Meeting*, pp. 956–962.

[6] S. Tamai, H. Sugimoto, and M. Yano, "Speed sensor-less vector control of induction motor with model reference adaptive system," in *Conf. Rec. 1987 IEEE-IAS Annu. Meeting*, pp. 189–195.

[7] A. Abbondanti and M. Brennen, "Variable speed induction motor drives use electronic slip calculator based on motor voltagesand currents," *IEEE Trans. Ind. Applicat.*, vol. IA-11, pp. 483–488, Sept./Oct. 1975.

[8] R. Venkataraman, B. Ramaswami, and J. Holtz, "Electronic analog slip calculator for induction motor drives," *IEEE Trans. Ind. Electron. Contr. Instrum.*, vol. IECI-27, pp. 110–116, Apr. 1980.

[9] D. Naunin, "Digital speed control of an induction motor with and without a speed sensor," in *Proc. 1983 Int. Power Electronics Conf.*, vol. 1, pp. 474–482.

[10] B. de Fornel, C. Reboulet, and M. Boidin, "Speed control by microprocessor for an induction machine fed by a static converter," in *Proc. 1984 IFAC Symp. Control in Power Electronics and Electrical Drives*, pp. 69–75.

[11] R. Joetten and G. Maeder, "Control methods for good dynamic performance induction motor drives based on current and voltages as measured quantities," *IEEE Trans. Ind. Applicat.*, vol. IA-19, pp. 356–363, May/June 1983.

[12] A. Nabae, I. Takashi, H. Akagi, and H. Nakano, "Inverter-fed induction motor drive systems with an instantaneous slip-frequency estimation circuit," in *Proc. IEEE Power Electronics Specialists Conf.*, 1982, pp. 322–327.

[13] R. Cuzner, R. Lorenz, and D. Novotny, "Application of nonlinear observers for rotor position detection on an induction motor using machine voltages and currents," in *Conf. Rec. 1990 IEEE-IAS Annu. Meeting*, pp. 416–421.

[14] A. Fitzgerald, C. Kingsley, and S. Umans, *Electric Machinery*. New York: McGraw-Hill, 1983.

[15] A. Bellini, A. de Carli, and M. L. Cava, "Parameter identification for induction motor simulation," *Automatica*, vol. 12, no. 4, pp. 383–386, July 1976.

[16] A. Consoli, L. Fortuna, and A. Gallo, "Induction motor identification by a microcomputer-based structure," *IEEE Trans. Ind. Electron.*, vol. IE-34, pp. 422–428, Nov. 1987.

[17] T. Irisa, S. Takata, R. Ueda, T. Sonoda, and T. Mochizuki, "A novel approach on parameter self-tuning method in ac servo system," in *Proc. 1984 IFAC Symp. Control in Power Electronics and Electrical Drives*, pp. 41–48.

[18] H. Kubota, K. Matsuse, and T. Fukao, "New control method of inverter-fed induction motor drive by using state observer with rotor resistance identification," in *Conf. Rec. 1985 IEEE-IAS Annu. Meeting*, pp. 601–606.

[19] D. Atkinson, P. Acarnley, and J. Finch, "Observers for induction motor state and parameter estimation," *IEEE Trans. Ind. Applicat.*, vol. 27, pp. 1119–1127, Nov./Dec. 1991.

[20] K. Hurst, T. Habetler, G. Griva, F. Profumo, and P. Jansen, "A self-tuning, closed-loop flux observer for sensorless torque control of standard induction machines," in *Proc. 1995 IEEE Power Electronics Specialists Conf.*, pp. 792–798.

[21] S. R. Sanders, "State estimation in induction machines," M.S. thesis, Dep. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, June 1985.

[22] M. Vélez-Reyes, "Decomposed algorithms for parameter estimation," Ph.D. dissertation, Dep. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, Sept. 1992.

[23] M. Vélez-Reyes, K. Minami, and G. C. Verghese, "Recursive speed and parameter estimation for induction machines," in *Conf. Rec. 1989 IEEE-IAS Annu. Meeting*, pp. 607–611.

[24] K. Minami, "Model-based speed and parameter tracking for induction machines," M.S. thesis, Dep. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, May 1989.

[25] K. R. Cho, "Detection of broken rotor bars in induction motors using parameter and state estimation," M.S. thesis, Dep. Elect. Eng. Comput.

Sci., Massachusetts Inst. Technol., Cambridge, June 1989.

[26] K. R. Cho, J. H. Lang, and S. D. Umans, "Detection of broken rotor bars in induction motors using parameter and state estimation," *IEEE Trans. Ind. Applicat.*, vol. 28, pp. 702–708, May/June 1992.

[27] M. S. Welsh, "Detection of broken rotor bars in induction motors using stator current measurements," M.S. thesis, Dep. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, May 1988.

[28] S. B. Leeb and J. L. Kirtley, "A multiscale transient event detector for nonintrusive load monitoring," in *Proc. 1993 IECON Int. Conf. Industrial Electronics, Control, and Instrumentation*, pp. 354–359.

[29] S. B. Leeb and J. L. Kirtley, "A transient event detector for nonintrusive load monitoring," U.S. Patent 5 483 153, Jan. 1996.

[30] S. R. Shaw, R. F. Lepard, and S. B. Leeb, "Desire: A power quality prediction system," in *Proc. North American Power Symp.*, Sept. 1996, pp. 581–586.

[31] P. C. Krause, *Analysis of Electric Machinery*. New York: McGraw-Hill, 1986.

[32] R. Johansson, *System Modeling and Identification*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[33] W. H. Press, S. A. Teukolsky, W. Vetterling, and B. Flannery, *Numerical Recipes in C*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 1992.

[34] S. R. Shaw, "Numerical methods for identification of induction motor parameters," M.S. thesis, Dep. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, Feb. 1997.

[35] A. V. Oppenheim and R. W. Schaefer, *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[36] R. Lepard, "Power quality prediction based on determination of supply impedance," M.S. thesis, Dep. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, May 1996.

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