APPLICATIONS OF A CLASS OF NONLINEAR FILTERS TO PROBLEMS IN POWER ELECTRONICS

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Abstract

We consider a class of nonlinear filters, particularly with regard to its usefulness in the power electronics field. This class of filters is characterized by the inclusion of a sorting element in the signal path. The sorting operation allows these filters to suppress impulsive noise while preserving edges and monotonic sections of signals. This introductory paper concentrates primarily on the median filter, it being the most accessible filter of the class. A working knowledge of issues arising in design and implementation is developed.

1. Introduction

Since the early 1970's there has been growing use of a class of discrete, nonlinear, and shift-invariant filters which incorporate a sorting element in their signal processing path. The inclusion of a ranking operation gives them abilities unavailable to linear filters, such as the capability to suppress impulse or transient noise from signals while preserving any underlying edges. These nonlinear smoothing properties coupled with ease of implementation have made such filters popular for many signal enhancement tasks in the fields of geophysical, biomedical, image, and radar signal processing. Present uses include smoothing and suppression of spike and other noise (known as speckle noise in images) [1, 2], edge detection [3–6], feature extraction, and signal coding [6, 7]. This growing group of applications has generated interest in a number of issues, ranging from the theoretical properties of such filters [8, 9] to fast VLSI implementations [10, 11], and has prompted us to consider their use in the field of power electronics. This introductory paper concentrates mainly on the median filter as being illustrative of these filters, though other elements of the class are presented briefly in Section 4.

In situations where signal and noise spectra occur in the same range (such as high frequency noise and "edgy" signals) and linear filters perform poorly, median type filters can provide a good alternative to linear smoothing. The superior performance of median (and more general rank-based) filters arises from the fact that linear filters are frequency oriented filters, shaping the spectra of signals, while median filters can be considered as geometrically oriented filters, shaping the local form of signals. For example, while the spectral content of impulses and steps are similar, they are geometrically different; steps are locally monotonic while impulses are not. In this terminology, median filters "pass" signals that are locally monotonic (or constant) and filter or smooth those that are not. We will make these notions precise in what follows.

2. The Median Filter

A median filter functions by sliding a symmetrically placed window across the data point by point and producing the median of the data in the window at the current time as output. This process is illustrated in Figure 1 for a window of size $2N+1$. Here $x(n+N)$ is the input signal and $y(n)$ is the output signal. The other filters of this class all share this moving window form. For the median we have:

$$y(n) = \text{median of } \{x(n-N), \ldots, x(n), \ldots, x(n+N)\} \quad (1)$$

For finite length signals, the beginning and end of the data are usually padded with the first and last value, respectively, as necessary to fill the window, though other

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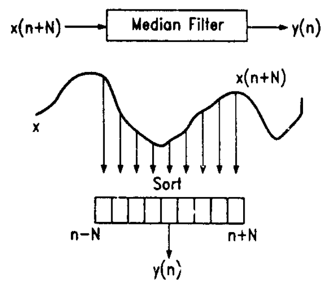


Figure 1: The median filter.

methods (e.g. padding with zeros) are possible. For convenience, we shall often refer to a median filter with window size $2N+1$ as a filter of size N.

To illustrate the effect of the median filter on signals in comparison with linear filters, Figure 2a shows an ideal square wave sequence, with amplitude values zero and five, corrupted by impulse (Cauchy) noise [12]. Figure 2b shows this signal after both median and fourth-order Butterworth lowpass filtering. The filter size for the median was set at $N=20$ points and the bandwidth of the digital Butterworth low pass set at $\omega = 0.15\pi$. Note how the median removes the spikes and effectively reconstructs the edgy signal. In contrast, the linear filter responds to the spikes as if they are impulses and fails to recover the underlying waveform.

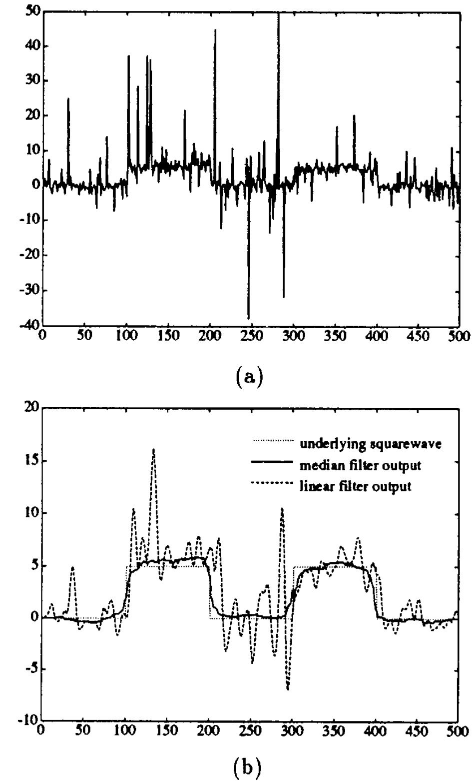


Figure 2: (a) Square wave corrupted with impulsive noise. (b) After median and Butterworth lowpass filtering.

Uses in Power Electronics

The class of nonlinear filters represented by the median filter promises to be valuable in the power electronics setting for removing spike or other impulse-like noise while preserving critical edges in waveforms. We discuss two applications of these filters: first, as an off-line tool for "cleaning up" experimental data, and second, as a real-time element for online monitoring and control applications.

It is often of interest to see how well simulated waveforms match such features as the rise times and slopes of measured data. In a simulation model, however, the parasitic components, such as MOSFET body capacitances, are not included for reasons of numerical efficiency. Therefore, simulated waveforms will not display such characteristic parasitic features as the ringing shown on the voltage step in Figure 3. The median filter is useful for removing such parasitic ringing from measured waveforms, along with spikes and hash, while preserving significant underlying features so that the pertinent characteristics of these waveforms may be compared with theoretical values.

To illustrate the potential of the median filter in a power electronics setting, consider the flyback converter

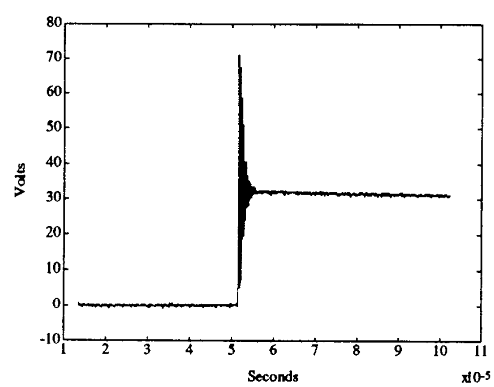


Figure 3: Switch voltage.

pass stage shown in Figure 4. This converter was operated in discontinuous conduction mode with a switching frequency of 5 kHz and is capable of delivering 25 watts through the 5 volt output winding. All displayed waveforms were sampled at 10 MHz with a digital storage

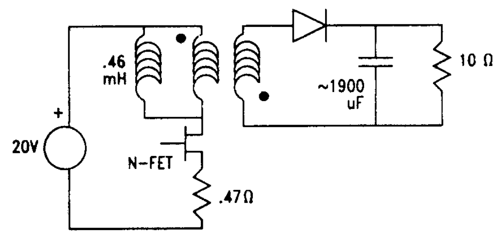


Figure 4: Flyback converter.

oscilloscope and processed off-line. For the experiments presented here, the pass stage was run open-loop. No effort was made to increase the robustness or general utility of the circuit. Its purpose was to provide, for the sake of illustration, examples of typical problems found in power electronic circuits.

Figure 3 shows the switch voltage when the controllable switch turns off. The spike on the rising edge of the switch voltage is caused by the MOSFET body capacitance ringing with the parasitic inductance in the transformer. Figure 5a shows the data after applying a median filter of size $N = 8$. Figure 5b shows the corresponding results of filtering the data in Figure 3 with three different fourth-order Butterworth filters whose cutoff frequencies span a range of values with respect to the sampling frequency. None of the analog filters is as capable of removing the parasitic ringing while preserving the step edge as the median filter. Figure 6 shows the voltage across the switch current sense resistor R1 when the controllable switch is closed. Again, note the ringing at the start and end of the ramp caused by the MOSFET body capacitance. The spike at the start of the ramp is particularly problematic in a control setting because it could cause pretriggering of the PWM latch in a current-mode controller. A median filtered version of the ramp ($N = 5$) is shown in Figure 7a. Shown in Figure 7b are the results of using three different fourth-order Butterworth filters on the data. The median filter is able to remove the spike while preserving the slope of the ramp.

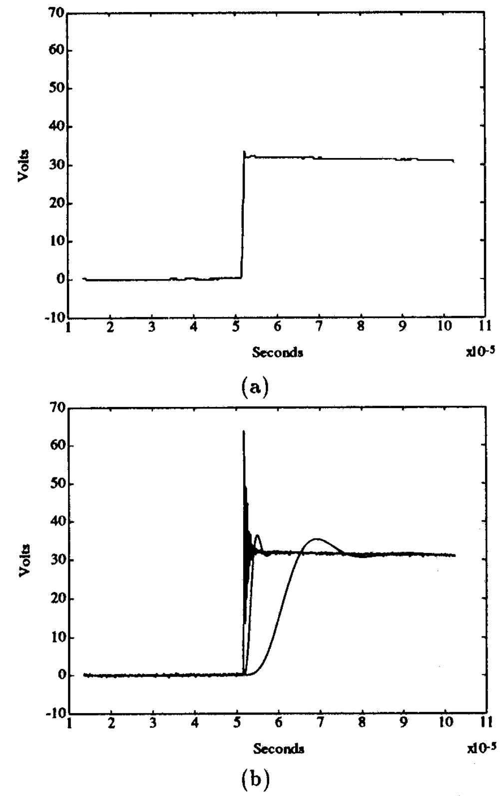


Figure 5: Filtered switch voltage: a) median, b) lowpass.

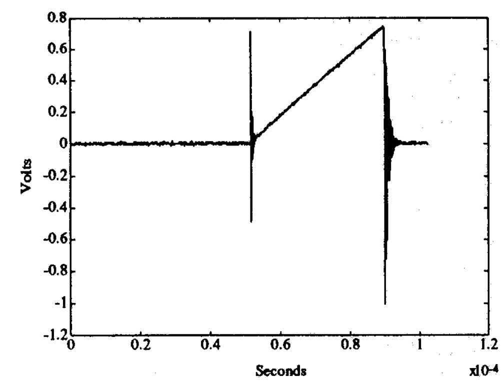
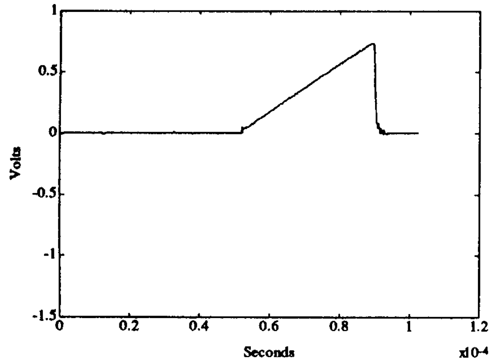


Figure 6: Switch current.

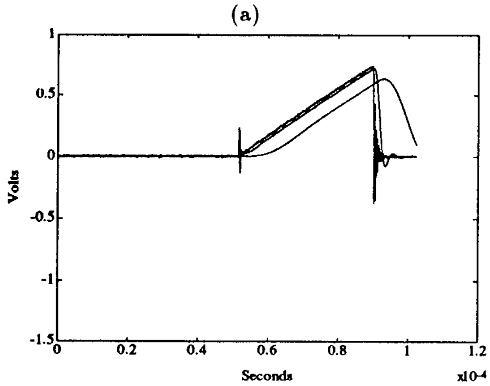
Real-time/Implementation Issues

Since the median filter requires only sorting, it is inherently immune to many of the pitfalls which must be avoided in digital implementations of linear filters, such as round-off error during floating-point mathematical operations. In applications where a median filter would be desirable, it would therefore also appear to have significant implementation advantages over linear filters. To investigate these expectations, we implemented a real-time median filter using a Texas Instruments TMS32020 digital signal processor.

The implementation revealed several drawbacks to the use of the current generation of commercial signal processors for real-time median filtering. The TMS32020, like most signal processors of its type, has an inherently serial architecture. As a result, the sorting process required to find the median window value is relatively time-



(a)



(b)

Figure 7: Filtered switch current: a) median, b) lowpass.

consuming especially for large window sizes. Even after carefully optimizing our computation strategy and algorithm, we found that the time required for this sorting operation limited operation of the filter to sample rates of 40 kHz at moderate window sizes ($N = 5$).

Our experience with the TMS32020 system has led us to explore more efficient ways of implementing median filters using both analog and digital hardware. Many custom architectures for implementing the median filter have appeared in the signal processing literature in recent years [10,11,13–22]. Most architectures are digital implementations which provide specialized hardware to perform fast ranking of the data in the filter window. These custom architectures, often operating in the video frequency range, are ideal for implementing median type filters, just as the fast floating-point multiply-accumulate units found in current digital signal processors are ideal for the implementation of linear filters. Discrete-time analog architectures optimized for the median filter have also been developed [23]. These developments suggest that general high performance sorting hardware might also have a place in future signal processor chips, just as fast floating-point multipication units have become essential to the present generation of digital signal processors. We will report the results of our experiments with such specialized sorting hardware at a later date.

3. Median Filter Properties

There are a number of ways to understand the functioning of the median filter and the broader class of nonlinear filters that are under consideration. These include interpretations based on stochastic [8], deterministic, and geometric concerns [9]. Historically, the median filter arose out of robust statistical considerations [25, 24], a route followed by other filters of the class (e.g. the M-filters [26], L-filters [27], and R-filters [28]). Here, we will focus on a geometric formalism to understand the properties of the median filter, as it is the most accessible for design and implementation purposes. This formalism will allow us to develop insights and intuitions similar to those for linear filters, including extensions of the concepts of bandwidth and invariant signals. Some of the other filters of the class containing a sorting element will be presented in Section 4.

Geometric Approach

From Figure 2 we can see qualitatively that the median filter smooths signals. This smoothing effect increases with the filter size N. However, median filters, being nonlinear, do not obey the superposition principle of linear filters, making discussion of frequency properties of limited value. Instead, we will think of the median filter in a shape oriented way [29]. Rather than viewing signals as composed of sinusoids (signals whose shape is unchanged by linear filtering), we shall present *geometric* structures whose shape is unchanged by the filters of interest as our signal building blocks. Such signals and structures that are invariant to a given filter (i.e. a fixed filter size N) are defined to be **roots** in the literature [9]. The example of steps and impulses being spectrally similar but geometrically different is one such case. We will show that steps are root structures of median filters while impulses are not. Concentrating on roots and their properties will thus focus our attention on fundamentally geometric aspects of signal structure.

In order to make these notions precise we will need some notation. We make the following definitions following [9]:

Constant Neighborhood: A section of at least $N+1$ consecutive points, all of which are identically valued.

Edge: A monotonically rising or falling region between two constant neighborhoods.

Impulse: A section of one to N points surrounded by identically valued constant neighborhoods whose boundary points are different from the constant neighborhoods.

Oscillation: Any section that is not part of a constant neighborhood, an edge, or an impulse.

Note that any signal may be decomposed into a series of the structures defined above, which may thus be considered as a set of geometric building blocks for signals. Also, our "ruler" for distinguishing different structures is of length N, and hence a function of the size of the filter under consideration. There is thus a direct relation between the size of a given filter and what is considered an edge, impulse, etc. We may now start our taxonomy of median filter and root properties following [9, 30].

Property 1 (Impulse Elimination) *Impulses are eliminated after a single pass of the median filter.*

This property formally states the impulse filtering ability of the median filter noted earlier. We may simply characterize all root signals and structures of the filter as follows:

Property 2 (Root Characterization) *A signal is a root of a median filter of size N if and only if the extended (padded) signal consists* only *of constant neighborhoods and edges.*

The above root characterization allows us to develop a notion of "geometric bandwidth" for the median filter, akin to the familiar frequency bandwidth used for linear filters.

Property 3 (Geometric Bandwidth) *In a root signal containing both increasing and decreasing regions, the sections of increase and decrease must be separated by a constant neighborhood (a section of at least $N+1$ identically valued points).*

Any root structure is therefore limited as to how quickly its *slope sign* may change, since a region of positive slope and negative slope must be separated by a constant region of at least $N+1$ points. There are no restrictions, however, on how quickly the *signal itself* may change (i.e the magnitude of the slope). This result explains why the median filter is effective at eliminating spike noise while preserving steps and monotone structures.

We can also relate the set of root structures at a given window size to those at another window size. Specifically, these root sets are nested as follows:

Property 4 (Root Nesting) *If a signal is a root of a median filter of size N, then it is also a root of a median filter of size $N-1$.*

This property may be viewed as a generalization of the linear concept of bandlimited signals. Here, the bandlimiting is in a geometrical sense, where the structures of a signal in the "passband" of a median filter of size N will pass unchanged through any filter of size $K < N$.

From a design standpoint Properties 3 and 4 serve as a guide for choosing a filter size N. For greater smoothing we want to choose a larger filter size, since more non-monotonic signal structures will appear as impulses and oscillations to the filter and be removed and reduced, respectively. On the other hand, to preserve signal structure of interest we cannot make N too large. Since the smallest signal structure passed by the filter will be only N+1 points long, we should choose N+1 no larger than the smallest signal structure we wish to preserve. For example, to eliminate the ringing in Figure 6 at the indicated sampling rate, a filter of size $N \geq 5$ is needed. Conversely, to preserve the peak structure of the ramp we want N as small as possible. These constraints represent the conflicting requirements of smoothing and resolution, as in linear filter design.

An important property of the median filter pertaining to root signals is that any finite-length signal, if repeatedly filtered (i.e. the output of one filtering used as the input to the next), will become a root in a finite number of passes.

Property 5 (Obtaining Roots) *Any nonroot signal (containing oscillations and impulses) of length L will become a root structure after at most $(L-2)/2$ successive filterings.*

In general, substantially fewer passes are needed to produce a root and certain variants of the standard median filter, such as the recursive median filter to be discussed below, produce roots in a single pass. The importance of this property is that structures, such as oscillations, which are not root structures and yet are *not* eliminated by a single median filter pass, can by removed by repeated filtering to a root (or by a filtering strategy that yields a root in one pass, as does the recursive median filter).

In summary, the above properties yield a shaped-based, geometric approach to the understanding and design of median filters. These insights and intuitions are parallels of the concepts used in linear filtering, such as bandwidth and invariant signals. Using these methods new filters can be designed with desired properties and the effects of existing filters can be analyzed.

4. Other Filters

As indicated throughout this work, the median is only one of a group of filters with similar noise suppression properties related by their use of an ordering element.

One problem with the median filter, and a motivation for examining other filters, is that the median filter often provides insufficient smoothing of *non*-impulsive noise. This problem is particularly acute in situations where the noise is basically well behaved (Gaussian) but contaminated by a "small" amount of impulsive noise. Some alternative filters within the same class are described next.

L-filters The L-filters are obtained by applying L-estimates on a moving basis. An L-estimate of a parameter is obtained as any *l*inear combination of the *ordered* data, where the weight of each data point depends only on its position in the ordered set. This ordered data set is know as the *order statistics* of the data [27]. An example L-estimate is the α-trimmed mean. A fraction α of the largest and smallest data values are deleted (weighted by zero) and the remaining values averaged (weighted by the inverse of their number). To create an L-filter, a moving window of data is obtained and sorted, as for the median filter, but now a linear combination of the sorted elements is produced as the output at each point. This operation is shown schematically in Figure 8a. The example α-trimmed mean L-estimate would yield the α-trimmed mean filter. Note that when $\alpha = 1/2$ the median filter is produced and when $\alpha = 0$ a simple moving average results. As another example, by choosing the second or third largest (the near maximum) value in the window, a peak detecting filter would be produced, but with less sensitivity to impulses than a true peak detector [27].

FIR-Median Hybrids This filter structure is shown in Figure 8b, and it can be seen to be something of the dual of the L-filter structure. For an L-filter, the windowed data are first sorted then FIR filtered, while for these filters the data are *first* FIR filtered in groups, then a median operation is performed. Advantages of these filters include greater potential noise reduction on linear portions of data and a larger class of root signals (including triangle waves) than for the median [31].

Recursive Median A straightforward generalization of the median filter that involves feedback is the recursive median filter [9]. It is obtained by using the most recent output as part of the filter input, as shown in Figure 8c. This simple modification produces a filter that yields a root signal after a single pass. The root obtained this way, however, will not in general be the same as one obtained after repeated ordinary median filtering. In fact, for a given size N, the effect of the recursive median will be greater than the corresponding ordinary median. This means it may be possible to use a smaller (thus faster) filter to achieve similar effects. Since the median operation involves choosing one of the input values for output, the output of this filter must be one of the original data points. Thus even though there is feedback from output to input, there is no stability issue, as with linear filters.

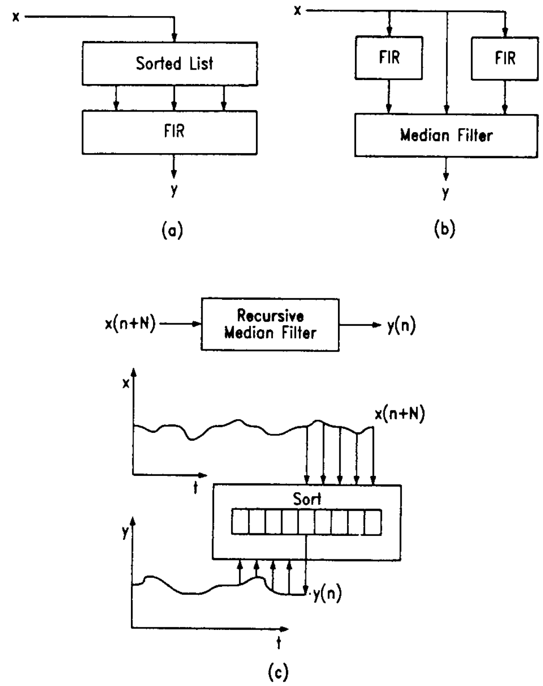


Figure 8: Other filters.

Adaptive Median Another variant of the median filter involves the use of adaptation on its size N. With the standard median filter we have the somewhat conflicting demands to make N large to increase smoothing but small to preserve resolution. One approach to this tradeoff has been to estimate the signal structure "on the fly" and then vary N based on this estimate as the filter window progresses. Where the signal appears relatively static, N is allowed to grow and where the signal appears to be changing, N is reduced. This approach attempts to optimize both smoothing and resolution in a *time-varying* solution. A recursive form was also examined and found to improve on the standard recursive median. Algorithms and hardware suggestions may be found in [32, 33].

Morphological Filters The family of morphological filters is so named because of their emphasis on affecting the shape of signals. The primitive operations that comprise the building blocks of these filters (erosion, dilation, opening, and closing) are obtained as a simple running maximum or minimum of the signal added to a translated kernel function. This translated "max/min of sums" operation is reminiscent of the translated "sum of products" operation of convolution, with the filter kernel serving the role of the impulse response. More complicated filters are obtained as compositions of these four primitive operations. A theory encompassing both these filters and linear filters and even the median has been developed but is beyond the scope of this work. We refer the interested reader to [29, 34, 35].

5. Conclusion

In this paper we have considered a class of nonlinear filters with potential application in power electronics. A main attribute of these filters, of which the most basic is the median filter, is their ability to remove impulsive noise while preserving edges. They are generally simple to implement off-line and should be included as an additional set of analysis tools for the power systems designer. Considerable work is currently underway to facilitate the use of these filters in real-time applications. The ideas presented in this paper should help the reader explore the properties and uses of this family of filters.

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