

# Build to Win: Electric Machines

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**Abstract**-- Electric machinery can serve as a powerful teaching tool in engineering curricula of all kinds. We have designed kits of flexibly configurable components that students use to quickly construct electromagnetic actuators and sensors. They can use these machines to verify design calculations, test their understanding of the machine operating principles, and to test power electronic drives that they also design and construct. In class, the properties of different motor designs and sizing can be related to commercial products, and commercial products can be used as inspiration for design contests in the teaching laboratory.

**Index Terms**--Electromagnetic actuators, motors, drives, power electronics, induction machine, permanent magnet machine, engineering education, power system components.

## I. ELECTRIC MACHINES: A PORTAL TO ENGINEERING DESIGN

OUR goal is to inspire engineering students to build to win. The latter half of the twentieth century saw marked changes in instruction for undergraduate engineering students. This period saw a laudable and transformative focus on engineering science in education, and a celebration of the invention and adaptation of modern computing in many forms. As we move forward, the value of engineering science and the pervasive application of computing will grow. A third challenge, managing and delighting in complexity, faces engineering students today. Many of our current engineering classes have been refined to focus on the beauty and technique of analytical methods and fundamental principles used in design, perhaps increasingly without reference to the practical physical systems for which these methods were developed. A balanced educational experience that combines a good appreciation of exciting, "information age" methods with the essential ability to manipulate and understand the physical world enables a student to design real systems [1]-[6].

As we experience the frisson of new electronic media and the contemplation of entirely new business models and delivery techniques for education, the distinction between training and education becomes acute [7]. Engineering educators are fundamentally in the business of building confidence. Students gain confidence and a joy in life-long learning by successfully tackling problems that demand craft, creativity, open-ended thinking, hypothesis generation, and the ability to modularize and "debug". The strongest learning experiences are often associated with a surprise, arguably best

found at the bench. Electric machines serve as outstanding focal points for teaching students to successfully engage complex design problems.

With notable exceptions, e.g., [1]-[3],[8],[9], the electric machine and electromagnetic actuators were in many instances "discarded with the bathwater" from our engineering curricula. Electric machines and drives offer industrially relevant, interdisciplinary design problems that challenge an engineer to think about electric circuits and electromagnetic, thermal, mechanical, and material design problems. The forces experienced between coils and magnets are practical magic, exciting to think about, and wondrous tools for building. We have used actuator, machine, and drive examples as engaging hands-on design problems at every level from K-12 to post-graduate students.

Our work in the classroom often begins with the interaction of a coil and a magnet. This leads to the consideration of more advanced electromagnetic machines and systems employing these machines. The paper proceeds with a look at these systems beginning with coils and magnets.

## II. FORCE AND ENERGY

Simple air-core coils can be wound by hand. They can also be purchased relatively inexpensively in a variety of sizes from not only specialized but also familiar vendors, e.g., [10]. The coils from [10] are nominally for speaker crossover circuits, but with great potential for other applications. We use coils like the one in [10] to introduce students to Lorentz forces, energy and co-energy, mechanics, and practical applications of electromagnetic actuators.

A basic two part "kit" for a student exploring the interactions of a coil and a magnet is shown in Fig. 1, with dimension  $x$  between the coil and magnet centers. The coil consists of  $N$  turns wound counter-clockwise as viewed from the top of the coil, where the magnet is located. Positive coil current results in magnetic flux pointing in the positive  $x$  direction, providing an opportunity for students to experiment with a magnetic compass, current probe, oscilloscope, power supply, and right-hand rule and data collection and handling techniques. The coil and magnet can exhibit several exciting behaviors, including attracting a magnet (e.g., suspended from a spring) to the coil energized with appropriate current, as in the case of a voice-coil actuator or speaker driver. With opposite current, the coil can repel or launch the magnet. Moving the magnet through the coil creates a generator.

Analysis of these behaviors begins with an understanding of the creation of magnetic flux in a coil. We might, for example, begin by having students consider the process of

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charging just an ideal coil, no magnet present yet, from a Norton equivalent source. After a long time, all of the Norton source current flows in the inductor, leaving zero volts across the inductor and the Norton source resistor. A finite total amount of energy, proportional to the inductance of the coil, is removed from the Norton current source. For a magnetically linear coil, this total energy removed from the source divides evenly: half of the energy is dissipated in the loss mechanism of the Norton source resistance, and half is stored in the magnetic field of the coil. This analysis presents an opportunity to explore the first law of thermodynamics and to review basic circuit analysis, differential equations for solving for the coil current, Thevenin and Norton equivalents, and the concepts of energy and co-energy. In the magnetically linear case, the energy stored in the inductor is balanced by the co-varying energy, or co-energy, dissipated in the resistor.

Introducing a magnet makes the system much more interesting. Reviewing the material properties and characteristics of magnetically "hard" materials, used in the creation of permanent magnets, leads to an opportunity to consider modeling. We use a Neodymium-Iron-Boron (NIB) magnet in the experiments to be described. As a first approximation, the NIB magnet can be reasonably well modeled as having an incremental permeability similar to air. In this case, a reasonable model for the magnet is another coil, driven by a current source. That is, the "coil and magnet" system shown in Fig. 1 can be approached as a "two coil" system, where the magnet in Fig. 1 is replaced by a second coil with current  $i_m$  at the magnet location, wound in the same sense as the lower coil. In this case, students consider the coupled system of two coils, each with a coil flux determined by the product of the self and mutual inductances and the coil currents. Mutual inductance  $M(x)$  is a function of separation between the two coils:

$$\begin{bmatrix} \lambda_c \\ \lambda_m \end{bmatrix} = \begin{bmatrix} L_c & M(x) \\ M(x) & L_m \end{bmatrix} \begin{bmatrix} i_c \\ i_m \end{bmatrix} \quad (1)$$

The forces between the two coils when both coils carry current can be computed by well-known methods using the derivative of either energy or co-energy with respect to position. Considering energy, the process of computing electromagnetic force between two coils energized beginning from initial rest involves inverting (1) to permit integration of current as a function of incremental flux, yielding energy. This expression for energy can then be differentiated to find force. Even for the simple two-coil system, this ultimately involves a dismal amount of algebra, which can be a character-building experience for a student to experience once.

This leads to a strong motivation to understand the concept of co-energy, which not only has important implications as part of the thermodynamic energy balance, but also permits computation of force while avoiding the need to invert and integrate constitutive laws like (1). In the thought process to find force from co-energy, the coil representing the magnet can be "energized" first, before the second (lower or launch) coil. This process stores an amount of energy in the

system that is not a function of position  $x$  in Fig. 1. The "magnet," or coil representing the magnet, is loaded with energy, analogous to an internal spring that is compressed and whose ends never experience further relative motion after initial compression or "charging" from zero amps to  $i_m = I$ .

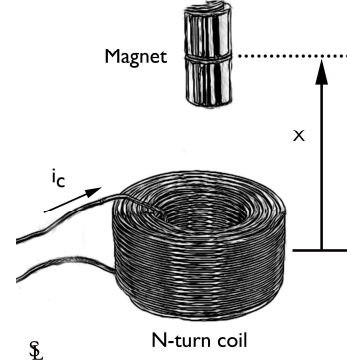


Fig. 1: Coil and magnet.

The system of Fig. 1 is particularly entertaining when the two components repel, that is, when  $i_m$  is negative. For purposes of computing force between the two components, a simplified flux-linkage constitutive law can focus on coil flux:

$$\lambda_c = L_c i_c + M(x)I = L_c i_c + \lambda_o(x) \quad (2)$$

Co-energy  $W'$  associated with charging the lower coil in Fig. 1 can be found by integrating (2) with respect to coil current  $i_c$  during the charging process that brings the lower coil current to  $I_c$ . The electromagnetic force,  $f$ , acting between the two coils (defined positive in the direction that increases the dimension  $x$  between the two coils) is:

$$f = \frac{\partial}{\partial x} W' = \frac{\partial}{\partial x} \int_0^{I_c} \lambda_c di_c \quad (3)$$

Assuming a "hard" magnet material with incremental permeability similar to air, or similarly assuming a "magnet" represented by an air-core coil, the inductance  $L_c$  is essentially independent of position  $x$ , and the force can be found by substituting (2) into (3):

$$f = I_c \frac{\partial \lambda_o}{\partial x} \quad (4)$$

This expression for force can in principle be used to solve for the motion of the magnet during a "launch" when the lower coil current is suddenly energized. As repelling magnetic fields force the magnet away from the coil, the electrical source energizing the lower coil puts energy into the mechanical system, the moving magnet projectile. This energy,  $E$ , can be expressed using the concept of mechanical power (force times velocity,  $v$ ), which relates back to the electromagnetic system through (4):

$$E = \int f * v dt = \int f \frac{dx}{dt} dt = \int i_c \frac{\partial \lambda_o}{\partial x} dx \quad (5)$$

The derivative of flux with respect to position has units of volt-seconds per meter. This quantity is analogous to the "back-EMF constant" or "motor constant" of a mechanically commutated dc motor, although for the linear actuator of Fig. (1), the back-EMF function in fact varies with position. This back-EMF function can be determined analytically, through fairly involved numerical calculations. The back-EMF function can also be determined empirically, which leads to a number of exciting opportunities to engage students at a bench in characterizing coils useful for different applications.

Figure 2 illustrates a student conducting measurements in preparation for characterizing coils that might be selected for use in a magnet thrower. The "drop tube" shown in Fig. 2 is carefully constructed to co-axially connect four coils, identical in the illustration, although the experiment could also be used to characterize different coils during the same "drop". Students measure and know the distance between the coil centers as shown in Fig. 2. They then drop a magnet (like that shown in Fig. 1) down the drop tube while recording the induced voltages. This leads to the beautiful traces shown in Fig. 3.

The top plot in Fig. 3 shows the four coil voltages captured during a magnet drop by a four channel oscilloscope monitoring all four coils, one per channel. The bottom plot in Fig. 3 shows the same experiment with the traces offset slightly to different ground positions to expose the separate behavior of each channel. The "Coil 1" trace is measured by a probe connected to the top coil in Fig. 2, progressing to the "Coil 4" trace shows the behavior of the bottom or fourth coil. The induced voltage in each coil shows the time rate of change of flux as the magnet enters the coil from the top. When the magnet and coil are centered, flux in the coil nulls and the induced voltage reverses as the magnet leaves on its way to the next coil in the drop tube. This experiment is a fabulous way to introduce Faraday's law. As the magnet accelerates down the tube, the time rate of change of flux for progressive coil traverses increases, with a concomitant increase in induced voltage. The position of the magnet can be determined at four points in time, as indicated for the magnet at the center of Coil 4 in the top trace of Fig. 3. The spacing between the traverses of the coil centers decreases as the magnet travels faster. A modern storage oscilloscope displays these four traces in color and in a snapshot capture, a display which usually draws audible acclaim.

The graphs in Fig. 3 are **not** the desired back-EMF function of the actuator, but rather the induced voltage, i.e., the time rate of change of flux, for each coil. To determine the change in flux versus a change in position, it is necessary to "scale" or re-parameterize the graphs of Fig. 3 using the velocity of the magnet dropped through the coil. This velocity can be determined using the time between the four "center points" illustrated in Fig. 3. We typically challenge students to reason out this point, and to use the data from Fig. 3, shown as circles in Fig. 4, to determine the magnet position as a function

of time. A computer mathematical assistant can be used to fit a curve of position versus time to the four points. This gives students a chance to see basic mechanics in action. The lead coefficient of the quadratic term in a second order fit to the data shown in Fig. 4 would ideally be half the acceleration due to gravity. Additional terms in the second order polynomial fit can arise due to friction and measurement error. A typical student curve fit is shown in the inset in Fig. 4, and plotted over the four data points from the drop experiment.



Fig. 2: Student completing preparatory measurements before dropping magnet through coils.

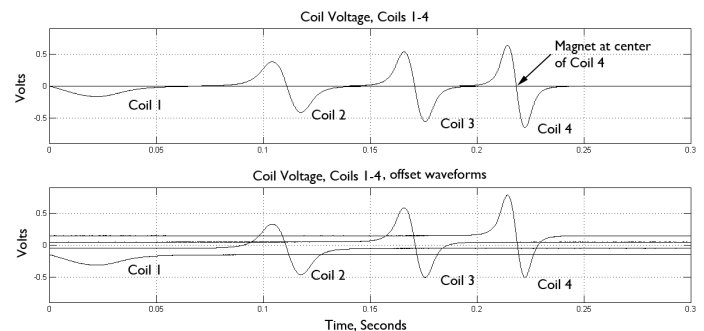


Fig. 3: Coil voltages during a magnet drop.

With the curve fit of Fig. 4, the second order polynomial expression for position can be differentiated to find velocity  $dx/dt$ . The voltage or time derivative of flux linkage illustrated in Fig. 3 can be divided by the velocity, or change in position versus time, to develop graphs of  $d\lambda/dx$  shown in Fig. 5.

The curves in Fig. 5 are satisfyingly repetitive, as would be expected from a drop tube with four identical coils: each coil should have the same back-EMF function. Small differences in the curves are discernable due to a variety of errors, including the effects due to friction and bumping in the drop tube. The careful design of this overall experiment can in itself become a terrific engineering problem for students, with both analytical and experimental components.

A single lobe of the curves illustrated in Fig. 5, shown in Fig. 6, is adequate to characterize the back-EMF function during a magnet launch, when a magnet is electromagnetically propelled from almost (but not quite) the center of a coil. The magnet must be offset slightly when inserted into the coil to avoid the null position where magnet north and south flux balance exactly in the coil. A typical student construction for

a magnet launcher is shown in Fig. 7, which guides the magnet with a Plexiglass tube and which constrains the initial position of the magnet inside the coil just slightly off of dead-centered. The associated circuit on the printed circuit board (PCB) is not required, but can be another set of design opportunities. The PCB shown in Fig. 7 helps prevent current from being applied to the coil for excessive durations, avoiding overheating. The circuit consists of a MOSFET, flyback diode for the coil, a MOSFET driver IC, and a 555-timer configured as a one-shot that applies a third-of-a-second pulse to the MOSFET gate when the "fire" switch (top left corner of Fig. 7) is actuated.

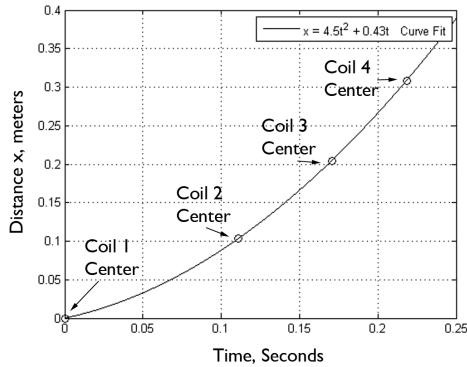


Fig. 4: Magnet position falling through drop tube.

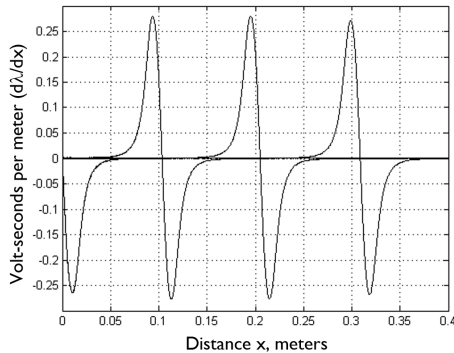


Fig. 5: Back-EMF function for all coils during fall.

Students can observe the coil current during a launch with a current probe, producing data like that shown in Fig. 8. This data exposes many features common to smaller electrical machines. For example, the "armature time constant" or rise time of the current is fast compared to the mechanical time constants in the system. Substantial motion of the magnet (in the region marked with an ellipse) occurs only after the coil current has fully developed from a voltage source.

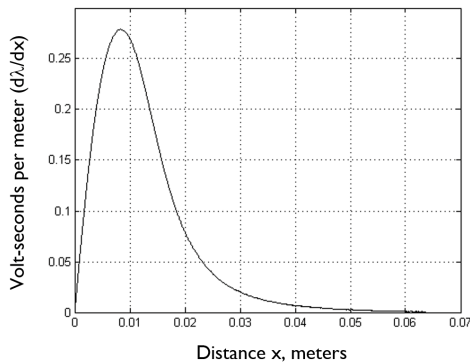


Fig. 6: Back-EMF from coil center.

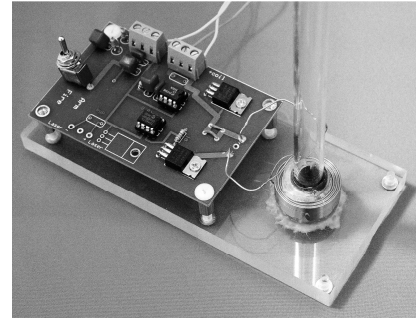


Fig. 7: Magnet launcher.

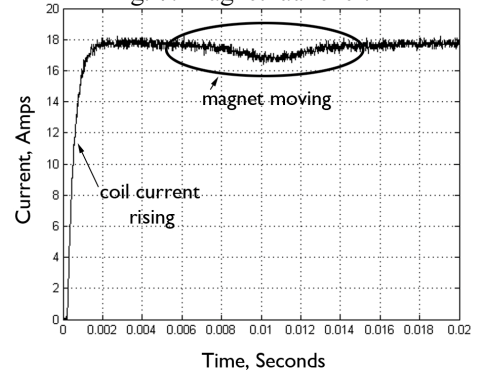


Fig. 8: Coil current during a launch.

Data like Figs. 6 and 8 can be used in (5) to estimate the mechanical energy imparted to the magnet, which manifests as kinetic energy during a launch. There are a variety of options for creating student design challenges and contests based on (5). Students could exploit the time-scale separation between the armature and mechanical time constants and estimate, with some error, the launch current as approximately equal to the source voltage divided by the coil resistance, assuming the launcher is not too efficient, i.e., that the speed voltage is relatively small. More accurately, students can use Fig. 8 as an opportunity to expose the essential role that back-EMF plays as an expression of the First Law of thermodynamics. The dip in current in Fig. 8 occurs because of the back-voltage from the coil - as the magnet moves, a "speed voltage" stands off the input source voltage and the input current drops. The speed voltage is the essential thermodynamic element that absorbs energy from the electrical system and transfers it to the mechanical system. Therefore, we might allow students to measure the Fig. 8 curve before a launch contest, but catch the magnet (plug the launch tube) during this testing to prevent students from estimating launch distance empirically. In this case, students can use the input current based on an observation like Fig. 8, which can be aligned to the back-EMF function shown in Fig. 6, and that the dip ends near the tail of the back-EMF function (around 0.06 meters in Fig. 6). With Fig. 6 and an estimate of launch current from Fig. 8, the launch velocity  $v$  can be computed by equating kinetic energy to  $E$  in (5); if a magnet with mass  $m$  is launched vertically against acceleration due to gravity  $g$ , the height  $h$  can be predicted by equating final potential energy to initial kinetic energy:

$$E = \frac{1}{2}mv^2 = m g h = \int i_c \frac{\delta\lambda_o}{\delta x} dx \quad (6)$$

A computer tool or even graph paper can be used to carry out the integral, using information from Fig. 8 to scale the curve in Figure 6, and then estimating the area under the scaled curve to determine initial launch velocity and final vertical height.

More complex design challenges are possible. An immediate variation is to launch the magnet at an angle as shown in Figure 9.

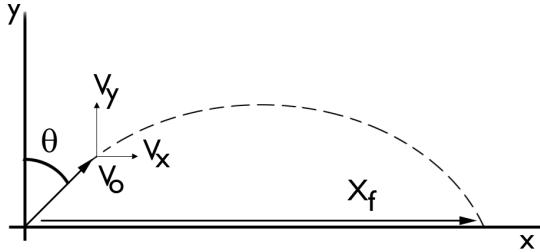


Fig. 9: Angled magnet launch.

In this case, the vertical component  $v_y$  of the initial velocity  $v_0$  determines the ultimate time of the overall flight. The horizontal component  $v_x$  of the initial velocity determines the distance flown. Initial velocity can still be estimated using (6), and the remainder of the problem is excellent for engaging younger students in considering basic mechanics. We have students conduct these calculations for launch contests, for example, to hit a target as illustrated in Fig. 10.



Fig. 10: Build to win.

The possibilities for further creative design and building with a magnet and coil are legion. For example, we have had students design and construct systems like the "shake flashlight" shown in Fig. 11. Shaking the tube bounces a magnet off springs attached to the white tube caps. As the magnet traverses the PVC tube, the coil generates voltage pulses like those shown in Fig. 3 which pass through a rectifier, charge a capacitor, and power an efficient LED. One design challenge for this exercise involves determining the optimal number of turns for the coil. The induced voltage grows essentially linearly with the number of turns. However, the relevant coil impedance quickly becomes a function of both the coil resistance and coil inductance as the number of turns increases. Coil impedance therefore grows nonlinearly, increasing rapidly with the number of turns and forcing an optimization peak that permits the largest induced voltage without limiting the available generated current too severely due to the coil impedance.

This activity also permits instruction in basic machining, soldering, and optics with parts available from the local hardware store. The coil and magnet system can also be used to create voice coil actuators or speaker drivers, e.g., [8],

which engage students in consideration of electrical and mechanical bandwidth for a desired speaker frequency response.

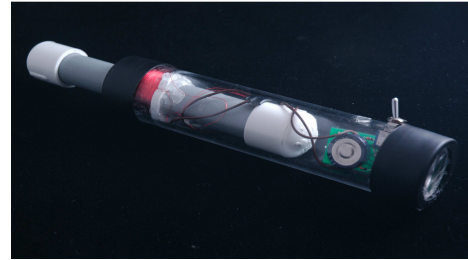


Fig. 11: Shake flashlight.

### III. ROTATING MACHINES

The dc motor is a superb example for introducing basic conservation laws, beginning circuit analysis, and basic electromechanical energy conversion. It is commercially and industrially relevant. Subtle aspects of dc machine construction and design, e.g., pole-face compensation and long-life brush design, can often be ignored in a first introduction. A basic commutator machine can be used to introduce and explain the right-hand rule and the basics of electromagnetic torque production to students at any level. We often begin with a review of a relevant commercial or industrial product or products, and use this review to pose a design problem. This makes the DC motor a perfect example for use in a "spiral" curriculum, where an example is reused in different classes, with different pedagogical emphasis. For example, we have used the DC motor in a sequence of four classes, first as an example of electromagnetic force in a transportation system to introduce energy conversion from electrical to mechanical forms, providing a specific example of the first law of thermodynamics. Next, in a later class, we have used the DC motor as an example for teaching machine shop practice, as students build their own motor. In later classes, the DC motor becomes a target for control by power electronics and microcontrollers. The spiral curriculum gives the students a chance to learn and absorb new ideas and skills while building on familiar knowledge.

A typical commercial machine example that we might use to motivate a "learn-design-build" activity is shown in Fig. 12. Figure 12 shows a 12-volt starter motor for an automobile. This particular machine has been cut open for display and examination in class. It normally has an almost completely sealed metal enclosure for protection from the environment. It includes a solenoid that throws the Bendix drive forward during starting to engage the internal combustion engine, a planetary reduction gear providing an r-to-1 reduction ratio, and a dc motor with a commutator arranged as "pie wedges" on an axial disk at the back of the machine. The dc motor is a permanent magnet machine with cylindrical magnets conformal to the starter case, and providing a air gap magnetic field of  $B$  Tesla that can be measured students using a flux probe. The mechanical construction and comparison to other motors, e.g., from vacuum cleaners and other appliances, are quickly discerned and lead to enthusiastic discussions about differences between the machines.

The field winding of a universal motor can be used as an early problem for learning about magnetic circuits. We have used magnetic circuits to motivate the study of circuit-solving techniques in general. Magnetic circuit analysis can also be used to discuss and compare a wound electromagnet with a permanent magnet, e.g., to understand the enormous energy density or effective “amp-turns-per-meter” provided by a contemporary neodymium high-performance magnet. These studies can lead to more detailed and subtle analysis of the results of the material properties. For example, a magnet can experience eddy currents due to the conductivity of magnetic material, and these losses can be compared to the eddy currents and hysteresis losses in a laminated wound-field yoke.

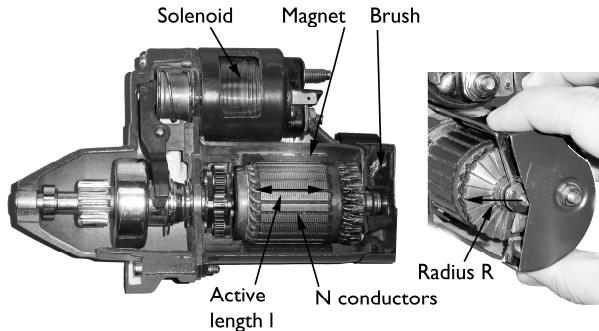


Fig. 12: Starter motor.

Surface wound machines provide practical example where the right-hand rule can predict the Lorentz force on the wires and therefore the torque of electromagnetic origin on the rotor. For more sophisticated students, we compare and contrast surface winding with the buried rotor wires similar to those on the starter motor in Fig. 12. For the “buried” wires, we have students solve or measure for the shielding effects of the rotor iron. This leads to an appreciation for the Maxwell stress tensor in computing traction on the rotor surface.

For a surface-wound machine with  $N$  conductors on the rotor surface with a rotor of radius  $R$ , active length  $l$ , and immersed in a radial magnetic flux density  $B$ , students can solve for the motor constant,

$$K = RNLB,$$

which, multiplied by the armature current  $i_a = i$ , is equal to the shaft torque of electromagnetic origin. With the motor constant  $K$  in hand, students can begin to understand how the motor performance is affected by changes in key physical design variables like the rotor radius, number of active turns of wire, and the active length of the machine. This leads to a beginning understanding of rudimentary machine-sizing rules for different applications. They can learn that doubling the active length of the machine can approximately double the shaft horsepower, assuming that the machine can spin smoothly and continue to function with its given rotor material and bearing system. They see, for example, the equivalence in some respects between doubling machine length and having two identical machines with front and rear shaft connections joined together. They begin to understand the importance of mechanical and thermal details in the machine, e.g., that it is not sufficient to simply double the length or radius of a rotor to increase shaft torque. They learn that it is also necessary to be

able to remove heat from a mechanically expanded system, and to support the system with smooth running bearings and a minimum of mechanical vibration.

A beginning understanding of the motor constant leads to many exciting lab and classroom experiments and demonstrations. For example, we challenge the students to develop a circuit model for a permanent-magnet brushed machine, and to use this model to understand the behavior of the machine driven by a voltage source versus a current source. The shaft of the machine can experience a load torque, for example proportional to the product of shaft speed  $\omega$  and a linear friction constant  $\beta$ .

The distinction between driving a dc machine with a current source versus a voltage source provide wonderful opportunities for understanding some limits of engineering approximation and for appreciating electromagnetic force and torque production. For example, we ask our students to energize a small dc machine in the laboratory with a fixed current of perhaps a quarter of an amp using a power supply configured as a current source. The power supply settles to whatever voltage is needed to drive the machine with a quarter of an amp, e.g., 12 volts for a typical small gear-head motor in our lab. They learn that current “programs” shaft torque, which they can check with a shaft bar and weight, or by grabbing the shaft with their fingers at different current levels. Then, we ask them to energize the machine with a voltage-source power supply running at 12 volts, allowing the machine to settle to steady operation at a quarter of an amp. Now, the students discover the inherent “feedback” loop present in the voltage-driven machine. When they grab the shaft, the machine slows fractionally. The back-EMF drops, and the machine draws more current, “fighting” the student in a manner very different from the fixed torque felt with the current source drive.

In our introductory feedback class, we use this experience to motivate modeling the machine with block diagrams. Fig. 13 shows typical student results. The block diagram on the top in Fig. 13 shows the model for a pm brushed dc motor driven with a current source. There is no inherent feedback loop that regulates the shaft speed based on the input current. Specifically, a change in the friction constant  $\beta$ , e.g., grabbing the shaft, perturbs the shaft speed. The block diagram on the bottom in Fig. 13, on the other hand, shows the model for the pm dc motor driven by a voltage source  $v_a$ . Changes in the friction constant are “buried” in a minor loop in the forward path of the machine model. The shaft speed is relatively insensitive to such changes if the motor is a “good” machine with relatively low armature impedance and, therefore, high “gain” in the forward path.

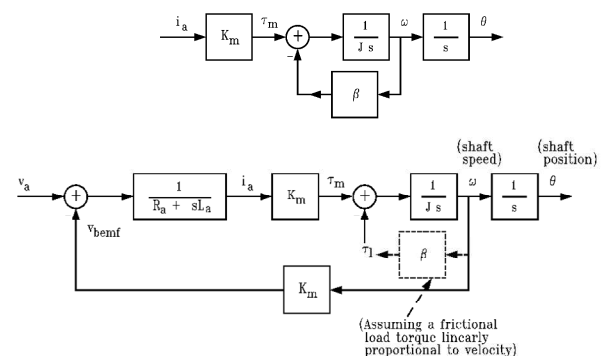


Fig. 13: Motor models with current (top) and voltage (bottom) drive.

We use this analysis and modeling to motivate “design and build” competitions at varying age levels and in varying courses and pedagogical venues. For example, in several classes, we challenge students to build a machine to match the specifications in the hand-held vacuum cleaner – a very difficult challenge rarely met in practice by students in the lab.

A “Dustbuster vacuum”-style dc machine spins at approximately 12,000 RPM (no load) from a 4.8 volt voltage source. At first, students attempting to design for high speed typically assume that they should design a motor with a **large** value for the motor constant,  $K$ . They reach this conclusion by assuming that “more  $K$  means more torque,” and more torque should push the shaft to higher speeds. This assumption is flawed in our vacuum-cleaner-based competition, where the input to the motor is a voltage source. Students then typically “rediscover” the voltage source model, but often apply it hastily, assuming that the armature impedance is negligible and that the machine is essentially a shaft-dependent back-EMF source. In this case, students may reach the conclusion that, since the armature voltage and back-EMF should approximately equilibrate,  $K$  should be as **small** as possible to maximize speed. At this point, they may be thoroughly confused and they have the opportunity to carefully revisit the lower machine model in Figure 13, with realistic loss mechanisms in place, i.e., a finite armature resistance and a linear shaft friction. In this case, students can solve for the steady-state shaft speed, finding that, in steady state:

$$\omega = \frac{K V}{K^2 + R_a \beta}$$

This is an interesting equation that illustrates an “optimum” point typical for many simple engineering “trade-off” problems. For a given set of losses (electrical resistance and mechanical friction), there is a “sweet spot” that maximizes kinetic energy stored in the rotor with respect to loss mechanisms in steady state. This can be seen in a typical plot of steady-state speed versus  $K$  (for  $V = 4.8$  volts, and an  $R_a \beta$  product arbitrarily chosen to be nine for illustration) as shown in Fig. 14.

Figure 14 is provocative in the classroom. There is typically no immediately obvious reason to a student why the armature resistance might be considered independently of  $K$ . The machine constant  $K$  includes length and number of conductors, and varying  $K$  should vary the armature resistance as well. As a practical matter, particularly for student machines where brushes might be made from spring steel, e.g., conveniently obtained paper clips, armature resistance is dominated by the brush resistance, particularly when the machine is warm in steady operation. In this case, the armature resistance might be considered independent of the machine constant  $K$ . Figure 14 summarizes the pedagogical essence of a number of very exciting design competitions for students studying motors and other energy conversion systems. For any given challenge application, students must understand the meaning of the motor constant  $K$  and how the physical parameters of the machine affect  $K$ . The key parameters - rotor radius, active machine length, field strength, and number of active conductors - are not completely independent

variables. The magnetic circuit of the machine is affected by the machine dimensions and the craft or skill that the students bring to assembling the machines. Many surprising and exciting competitions can occur when elements of craft and skill are mixed with a complete understanding of the basic physical principles behind the machine. We use the trade-offs inherent in the discussion of Fig. 14 to sharpen students' analysis skills. We ask them to determine by “back-of-the-envelope” methods if they expect a machine they might construct to be a “small  $K$ ” or “big  $K$ ” machine, i.e., likely to be on one side or the other of the peak indicated in Fig. 14. We have students consider the meaning of “small” and “large”  $K$  values, and the limits implied by these regimes, i.e., insufficient ability to produce torque (small  $K$ ) or limited ability to push current into the machine with a fixed input voltage (large  $K$ ). This leads them to consider moment of inertia to find  $J$ , use this estimate of  $J$  to guess at the friction coefficient  $\beta$  with data from timing a “spin down” of a sample rotor in a lecture demonstration, and then to estimate upper bounds on  $K$  by guessing at field strengths, likely number of turns to fit on a rotor, rotor radius, and so forth. In activities where we chose to avoid the use of flux-focusing ferromagnetic materials to ease machining requirements, motors typically are found to be likely to be “small  $K$ ”, and the race is on to make a motor with the largest possible  $K$  and the fastest steady state speed (in a speed contest) by approaching the peak in Fig. 14 from the left side of the graph.

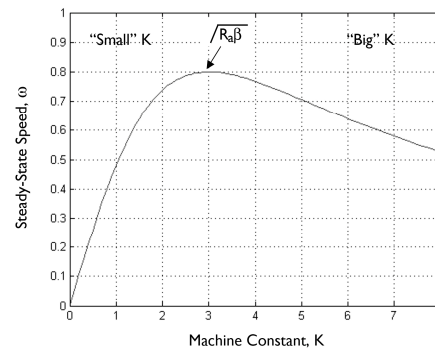


Fig. 14: Steady state speed versus motor constant.

Typical results during a contest are shown in Fig. 15, which illustrates another important point for students to consider, typically after a first contest. The peak in Fig. 14 implies operating conditions for the motor that are only 50% efficient - with losses clearly and dramatically indicated during the speed contest shown in Fig. 15. These results open the door to new contests and design considerations for efficient machines that achieve necessary mechanical goals.

These design trade-offs vis-à-vis the machine constant  $K$  are not limited to competitions in which the students are actually designing the machine. We have used this analysis with students in other courses and majors, for example, in mechanical engineering. These students may be involved in classes where a fixed, known motor is bought for every student for some application under consideration in the course. Intriguingly, gear ratio,  $r$ , between the motor shaft and a wheel or other mechanical load affects  $K$  directly. That is, students can work with a new variable,  $K_{\text{mod}} = rK$ , and produce a plot just like Fig. 14 for steady-state speed versus  $K_{\text{mod}}$ .

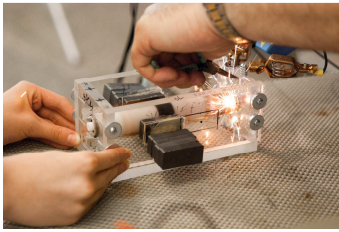


Fig. 15: A fifty percent efficient motor (maybe).

We have used these ideas to develop teaching modules and engineering design competitions for students at almost every age level. For quick exposures that still involve design variability and touching real tools, we have built a motor “erector set” kit [11], with a variety of different blocks and rotors, that students can use to assemble a motor of their choice within some design limitations. A typical example is shown in Figure 16.

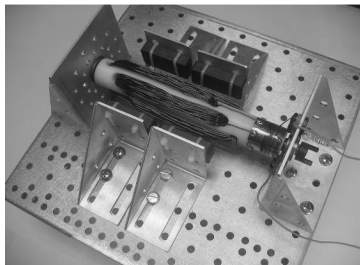


Figure 16: Motor “erector set”.

We have also developed a flexible, multi-use, 3-phase axial-flux machine suitable for laboratory instruction [12,13]. This machine is configurable as a permanent magnet (brushless dc) motor or as an induction machine. It would typically be used with a three-phase inverter built by our students as part of a power electronics laboratory. The machine is shown in [12], and can be configured as an induction machine, with copper disks secured to a steel rotor backing. The steel disks can also be “flipped” to bring magnets and a steel magnetic circuit facing the stator.

#### IV. A LITTLE, OFTEN

We have designed special kits of parts that can permit students to quickly wind and construct dc motors, pm machines, and induction motors, and then to use these motors in power electronic drives that the students also design and construct. These machines are unconventional compared, for example, to the radial-magnetic flux machines most common in industrial environments. However, they give undergraduate students a “hands-on,” immediate feel for simple motor sizing rules, electrical terminal models, drive efficiency, and the interaction between a motor and a power electronic drive. More generally, electromagnetic machines provide engaging examples for introducing engineering topics and teaching students to manage complexity methodically. All classes, including programming, algorithm, and math courses, could benefit from occasional consideration of electric machines.

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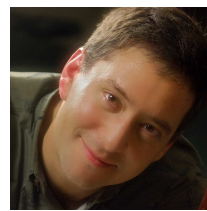
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#### VI. BIOGRAPHIES



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